Optical Sciences PhD Qualifying Exam:
Instructions and Equation Sheet

Answer all four questions on the following pages. Start each answer on a new sheet of paper.

In the upper right hand corner of each sheet, write your code number (NO NAMES PLEASE) and the problem number (course number) that you are answering. Attach together all sheets for a given problem in order.

When finished, insert your answers into the envelope provided. Also insert all scratch paper, this equation sheet, the exam problems, and any other items that were distributed to you in the envelope.

The following are some helpful items:

$h = 6.625 \times 10^{-34}\ \mathrm{J \cdot s} = 4.134 \times 10^{-15}\ \mathrm{eV \cdot s}$

$e = 1.6 \times 10^{-19}\ \mathrm{C}$

$c = 3.0 \times 10^{8}\ \mathrm{m/s}$

$k_B = 1.38 \times 10^{-23}\ \mathrm{J/K}$

$\sigma = 5.67 \times 10^{-8}\ \mathrm{W/K^4 \cdot m^2}$

$\epsilon_0 = 8.85 \times 10^{-12}\ \mathrm{F/m}$

$\mu_0 = 1.26 \times 10^{-6}\ \mathrm{H/m}$

$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$

$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$

$2\cos A \cos B = \cos(A-B) + \cos(A+B)$

$2\sin A \sin B = \cos(A-B) - \cos(A+B)$

$2\sin A \cos B = \sin(A+B) + \sin(A-B)$

$2\cos A \sin B = \sin(A+B) - \sin(A-B)$

$\sin 2A = 2\sin A \cos A$

$\cos 2A = 2\cos^2 A - 1$

$\cos 2A = 1 - 2\sin^2 A$

$\sin^2(\frac{A}{2}) = \frac{1}{2}(1 - \cos A)$

$\cos^2(\frac{A}{2}) = \frac{1}{2}(1 + \cos A)$

$\sinh x = \frac{1}{2}(e^x - e^{-x})$

$\cosh x = \frac{1}{2}(e^x + e^{-x})$

$\nabla(\phi + \psi) = \nabla\phi + \nabla\psi$

$\nabla\phi\psi = \phi\nabla\psi + \psi\nabla\phi$

$\nabla \cdot (\mathbf{F} + \mathbf{G}) = \nabla \cdot \mathbf{F} + \nabla \cdot \mathbf{G}$

$\nabla \times (\mathbf{F} + \mathbf{G}) = \nabla \times \mathbf{F} + \nabla \times \mathbf{G}$

$\nabla(\mathbf{F} \cdot \mathbf{G}) = (\mathbf{F} \cdot \nabla)\mathbf{G} + (\mathbf{G} \cdot \nabla)\mathbf{F} + \mathbf{F} \times (\nabla \times \mathbf{G}) + \mathbf{G} \times (\nabla \times \mathbf{F})$

$\nabla \cdot (\phi\mathbf{F}) = \phi(\nabla \cdot \mathbf{F}) + \mathbf{F} \cdot \nabla\phi$

$\nabla \cdot (\mathbf{F} \times \mathbf{G}) = \mathbf{G} \cdot \nabla \times \mathbf{F} - \mathbf{F} \cdot \nabla \times \mathbf{G}$

$\nabla \cdot (\nabla \times \mathbf{F}) = 0$

$\nabla \times (\phi\mathbf{F}) = \phi(\nabla \times \mathbf{F}) + \nabla\phi \times \mathbf{F}$

$\nabla \times (\mathbf{F} \times \mathbf{G}) = \mathbf{F}(\nabla \cdot \mathbf{G}) - \mathbf{G}(\nabla \cdot \mathbf{F}) + (\mathbf{G} \cdot \nabla)\mathbf{F} - (\mathbf{F} \cdot \nabla)\mathbf{G}$

$\nabla \times (\nabla \times \mathbf{F}) = \nabla(\nabla \cdot \mathbf{F}) - \nabla^2\mathbf{F}$

$\nabla \times \nabla\phi = 0$

$\oint_{\mathrm{S}}(\mathbf{F} \cdot \mathbf{n})\, da = \int_{\mathrm{V}}(\nabla \cdot \mathbf{F})\, d^3x$

$\oint_{\mathrm{C}} \mathbf{F} \cdot d\ell = \int_{\mathrm{S}}(\nabla \times \mathbf{F}) \cdot \mathbf{n}\, da$

$\oint_{\mathrm{S}} \phi\mathbf{n}\, da = \int_{\mathrm{V}} \nabla\phi\, d^3x$

$\oint_{\mathrm{S}} \mathbf{F}(\mathbf{G} \cdot \mathbf{n})\, da = \int_{\mathrm{V}}[\mathbf{F}(\nabla \cdot \mathbf{G}) + (\mathbf{G} \cdot \nabla)\mathbf{F}]\, d^3x$

$\oint_{\mathrm{S}}(\mathbf{n} \times \mathbf{F})\, da = \int_{\mathrm{V}}(\nabla \times \mathbf{F})\, d^3x$

PhD Qualifying Exam, August 2025

Opti 501, Day 1

<u>System of units: SI (or MKSA)</u>

a) Write Maxwell's differential equations in their most complete form, including contributions from free-charge and free-current densities, as well as those from polarization and magnetization sources. Explain the meaning of each symbol that appears in these equations.

b) Derive the charge-current continuity equation directly from Maxwell's equations, and explain the meaning of this equation. Be brief but precise.

c) Define the bound-electric-charge and bound-electric-current densities. Use these entities to eliminate the $\boldsymbol{D}$ and $\boldsymbol{H}$ fields from Maxwell's equations. (In other words, rewrite Maxwell's equations with the help of bound-charge and bound-current densities in such a way that only the $\boldsymbol{E}$ and $\boldsymbol{B}$ fields would appear in the equations.)

d) Show that the bound-charge and bound-current densities of part (c) satisfy their own charge-current continuity equation.

---

1) You are using a digital camera to take a picture of a house against a distant mountain backdrop. The camera is focused on the house. You have a wide selection of camera lenses and 16:9 (width = 12.48mm, height = 7.02 mm) CCD detectors at your disposal. The CCD detectors available to you have pixels with sizes ranging between 3 μm and 6 μm, however, cameras with pixel sizes of exactly 3 μm and 6 μm are not available. You want to make sure that you can take a high-quality picture of the house and the mountains and therefore, want to achieve a diffraction-limited image based on the pixel pitch of the CCD detector you selected. The location allows you to place the camera anywhere between 10 and 30 meters (not exactly 10 nor 30 m) away from the house. Assume that the lens is a thin lens with the stop at the lens.

Select a CCD detector and a location to place your camera and determine:

a) (5 pts) What are the hyperfocal distance and acceptable blur for your camera system?

b) (10 pts) Determine the focal length, *f*, and the diameter, *D*, of the lens.

c) (5 points) How far behind the lens must the detector be located? What is the maximum size (height and width) of the house in meters you would be able to image on the detector?

d) (5 points) What is the horizontal and vertical Field of View of this photographic system (in degrees)?

e) (5 points) What is the f/# of this camera? What is the closest object that will be considered to be in focus?

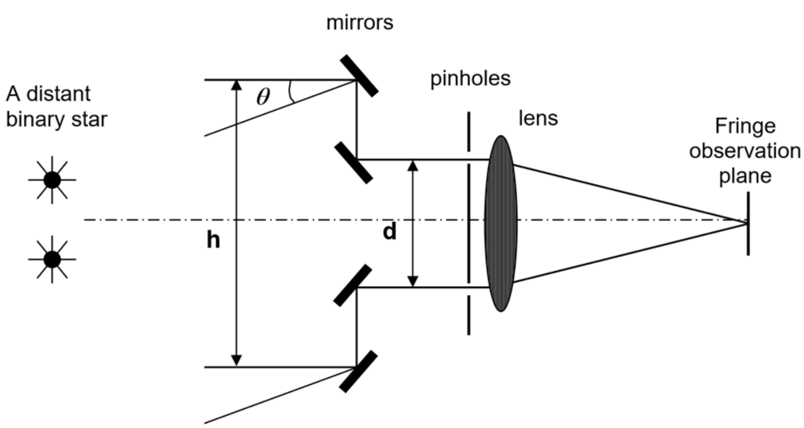
f) (10 points) It is an overcast day (outdoor illuminance = 10^3 lux = 10^3 lm/m^2). The reflectance of the house is $\rho = 0.5$ and the mountains have an average reflectance of $\rho = 0.18$. What are the image plane illuminances for the house and the mountains individually?

OPTI505 Day 1 (FALL 2025)

The Michelson Stellar interferometer shown below is used to measure the separation of a particular binary star. The radiance distribution is

$$M_R(\theta) = I_0\left\{\delta(\theta+\Delta\theta)+\delta(\theta-\Delta\theta)\right\},$$

where +/- $\Delta\theta$ are the angles between the optical axis and the ray coming from each star, and I_0 is the radiance of each star. The binary star separation is given in terms of $\Delta\theta = L/2z_{src}$, where L is the physical separation of the stars in length and z_{src} is the distance from the stars to the interferometer. The angle $\theta = y_{src}/z_{src}$. Assume that a narrow-band optical filter is used to obtain quasimonochromatic illumination at $\bar{\lambda}$ = 500 nm. The first zero in fringe visibility occurs at a mirror separation of h = 15 meters.



a) (25%) Write the radiance distribution in terms of physical lengths L and z_{src}.
b) (25%) Derive an expression for the spatial coherence visibility function $\mu_{12}^x(\theta_{ph}/\lambda)$. [Hint: Normalization of M_R can be done by setting $\mu_{12}^x(0)$ = 1.]
c) (50%) Determine $\Delta\theta$ by solving (b) under the conditions specified in the problem.

You may find the following Fourier Transform pairs useful:

$$\cos(2\pi\xi_0 x) \rightarrow \frac{1}{2}\left[\delta(\xi+\xi_0)+\delta(\xi-\xi_0)\right]$$

$$\frac{1}{2}\left[\delta(x+x_0)+\delta(x-x_0)\right] \rightarrow \cos(2\pi\xi x_o)$$

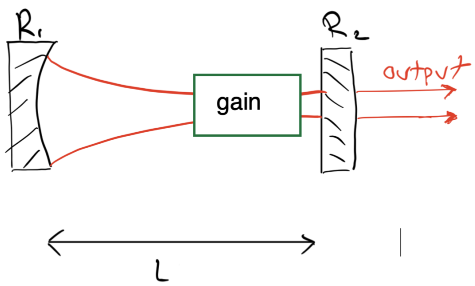
$$\delta(x \pm x_0) \rightarrow e^{\pm j2\pi x_0 \xi}$$

$$e^{\pm j2\pi x\xi_o} \rightarrow \delta(\xi \mp \xi_0)$$

2025 Qualifying Exam
OPTI 511R / 570 / 544
(10 points total)

A laser system operates at a wavelength of $\lambda_0 = 1064\,\text{nm}$. The 5 cm long gain medium is placed inside a linear optical cavity of length $L = 30\,\text{cm}$ between two mirrors. Mirror 1 has a reflectivity $R_1 = 99.9\%$ and radius of curvature $ROC_1 = 80\text{cm}$, and mirror 2 (the output coupler) has a reflectivity $R_2 = 95\%$ and an infinite radius of curvature (i.e. a flat mirror).

The gain cross-section at resonance is $\sigma(\nu_0) = 2.5 \times 10^{-19}\,\text{cm}^2$, and the saturation intensity is $I_{\text{sat}} = 10\,\text{kW/cm}^2$. The total inversion density is $\Delta N_0 = 5 \times 10^{18}\,\text{cm}^{-3}$.



Answer the following questions:

(a) Calculate the **small-signal gain coefficient** g_0 (in cm^{-1}) of the laser medium.

(b) Find the **round-trip gain** and **round-trip loss** for the laser cavity. Express both quantities and determine whether lasing is possible.

(c) Determine the **threshold inversion density** ΔN_{th} required for lasing.

(d) If the laser is pumped to twice the threshold inversion density, what will be the **steady-state intracavity on-axis intensity** I (in W/cm^2) inside the gain medium, assuming a saturation model:

$$g(I) = \frac{g_0}{1 + I/I_{\text{sat}}}.$$

For this part of the problem, you can assume that the beam diameter does not significantly change within the gain medium.

(e) What is the spot size (radius w, not diameter) of the laser beam on the output coupler? Over approximately what distance beyond the output coupler will the laser beam stay collimated? Justify your answer.

Recall that for a Gaussian laser beam, the wavefront radius of curvature is given by:
$R(z) = z + z_o^2/z$

Optical Sciences PhD Qualifying Exam:
Instructions and Equation Sheet

Answer all four questions on the following pages. Start each answer on a new sheet of paper.

In the upper right hand corner of each sheet, write your code number (NO NAMES PLEASE) and the problem number (course number) that you are answering. Attach together all sheets for a given problem in order.

When finished, insert your answers into the envelope provided. Also insert all scratch paper, this equation sheet, the exam problems, and any other items that were distributed to you in the envelope.

The following are some helpful items:

$h = 6.625 \times 10^{-34}\ \mathrm{J \cdot s} = 4.134 \times 10^{-15}\ \mathrm{eV \cdot s}$

$e = 1.6 \times 10^{-19}\ \mathrm{C}$

$c = 3.0 \times 10^{8}\ \mathrm{m/s}$

$k_B = 1.38 \times 10^{-23}\ \mathrm{J/K}$

$\sigma = 5.67 \times 10^{-8}\ \mathrm{W/K^4 \cdot m^2}$

$\epsilon_0 = 8.85 \times 10^{-12}\ \mathrm{F/m}$

$\mu_0 = 1.26 \times 10^{-6}\ \mathrm{H/m}$

$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$

$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$

$2 \cos A \cos B = \cos(A - B) + \cos(A + B)$

$2 \sin A \sin B = \cos(A - B) - \cos(A + B)$

$2 \sin A \cos B = \sin(A + B) + \sin(A - B)$

$2 \cos A \sin B = \sin(A + B) - \sin(A - B)$

$\sin 2A = 2 \sin A \cos A$

$\cos 2A = 2 \cos^2 A - 1$

$\cos 2A = 1 - 2 \sin^2 A$

$\sin^2(\frac{A}{2}) = \frac{1}{2}(1 - \cos A)$

$\cos^2(\frac{A}{2}) = \frac{1}{2}(1 + \cos A)$

$\sinh x = \frac{1}{2}(e^x - e^{-x})$

$\cosh x = \frac{1}{2}(e^x + e^{-x})$

$\nabla(\phi + \psi) = \nabla\phi + \nabla\psi$

$\nabla\phi\psi = \phi\nabla\psi + \psi\nabla\phi$

$\nabla \cdot (\mathbf{F} + \mathbf{G}) = \nabla \cdot \mathbf{F} + \nabla \cdot \mathbf{G}$

$\nabla \times (\mathbf{F} + \mathbf{G}) = \nabla \times \mathbf{F} + \nabla \times \mathbf{G}$

$\nabla(\mathbf{F} \cdot \mathbf{G}) = (\mathbf{F} \cdot \nabla)\mathbf{G} + (\mathbf{G} \cdot \nabla)\mathbf{F} + \mathbf{F} \times (\nabla \times \mathbf{G}) + \mathbf{G} \times (\nabla \times \mathbf{F})$

$\nabla \cdot (\phi\mathbf{F}) = \phi(\nabla \cdot \mathbf{F}) + \mathbf{F} \cdot \nabla\phi$

$\nabla \cdot (\mathbf{F} \times \mathbf{G}) = \mathbf{G} \cdot \nabla \times \mathbf{F} - \mathbf{F} \cdot \nabla \times \mathbf{G}$

$\nabla \cdot (\nabla \times \mathbf{F}) = 0$

$\nabla \times (\phi\mathbf{F}) = \phi(\nabla \times \mathbf{F}) + \nabla\phi \times \mathbf{F}$

$\nabla \times (\mathbf{F} \times \mathbf{G}) = \mathbf{F}(\nabla \cdot \mathbf{G}) - \mathbf{G}(\nabla \cdot \mathbf{F}) + (\mathbf{G} \cdot \nabla)\mathbf{F} - (\mathbf{F} \cdot \nabla)\mathbf{G}$

$\nabla \times (\nabla \times \mathbf{F}) = \nabla(\nabla \cdot \mathbf{F}) - \nabla^2\mathbf{F}$

$\nabla \times \nabla\phi = 0$

$\oint_{\mathrm{S}} (\mathbf{F} \cdot \mathbf{n})\, da = \int_{\mathrm{V}} (\nabla \cdot \mathbf{F})\, d^3x$

$\oint_{\mathrm{C}} \mathbf{F} \cdot d\ell = \int_{\mathrm{S}} (\nabla \times \mathbf{F}) \cdot \mathbf{n}\, da$

$\oint_{\mathrm{S}} \phi\mathbf{n}\, da = \int_{\mathrm{V}} \nabla\phi\, d^3x$

$\oint_{\mathrm{S}} \mathbf{F}(\mathbf{G} \cdot \mathbf{n})\, da = \int_{\mathrm{V}} [\mathbf{F}(\nabla \cdot \mathbf{G}) + (\mathbf{G} \cdot \nabla)\mathbf{F}]\, d^3x$

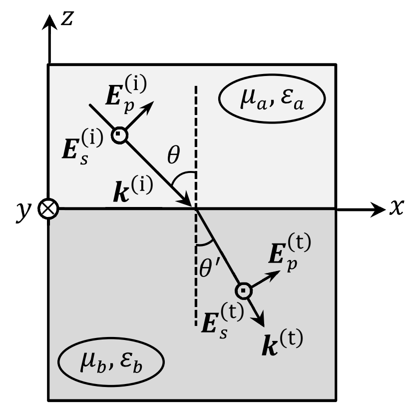
$\oint_{\mathrm{S}} (\mathbf{n} \times \mathbf{F})\, da = \int_{\mathrm{V}} (\nabla \times \mathbf{F})\, d^3x$

PhD Qualifying Exam, August 2025

Opti 501, Day 2

<u>System of units: SI (or MKSA)</u>

Consider the flat interface between two linear, isotropic, homogeneous media specified by their relative permeability and permittivity at the incidence frequency, namely, (μ_a, ε_a) for the medium above, and (μ_b, ε_b) for the medium below the interface. These material parameters (i.e., $\mu_a, \mu_b, \varepsilon_a, \varepsilon_b$) are assumed to be real-valued and positive. A homogeneous plane-wave of frequency ω arrives at the interfacial xy-plane; the plane of incidence is xz, the incidence angle is θ, and the E-field components of the incident beam are $\boldsymbol{E}_p^{(\mathrm{i})}$ and $\boldsymbol{E}_s^{(\mathrm{i})}$, as indicated in the figure. The E-field components of the transmitted beam, also a homogeneous plane-wave, are $\boldsymbol{E}_p^{(\mathrm{t})}$ and $\boldsymbol{E}_s^{(\mathrm{t})}$, and the angle between the transmitted k-vector and the surface-normal is θ', as shown.



a) Invoking the dispersion relation $\boldsymbol{k} \cdot \boldsymbol{k} = (\omega/c)^2 \mu(\omega)\varepsilon(\omega)$, write expressions for the k-vectors of the incident and transmitted plane-waves shown in the above figure. ($c = 1/\sqrt{\mu_0 \varepsilon_0}$ is the speed of light in vacuum.)

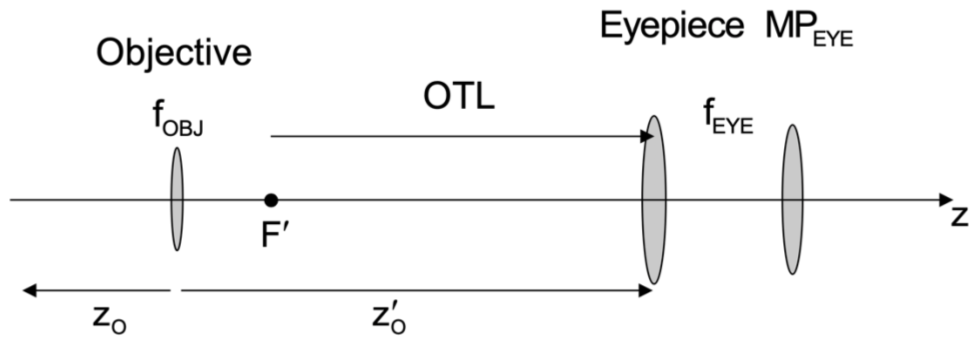
b) Invoking Maxwell's boundary conditions, explain why the transmitted wave has the same frequency ω as the incident wave. What do these boundary conditions reveal about the relation between θ and θ'?

c) Use Maxwell's third equation, $\boldsymbol{\nabla} \times \boldsymbol{E} = -\partial \boldsymbol{B}/\partial t$, to determine both the incident and the transmitted H-field components. (As usual, $\boldsymbol{B} = \mu_0 \mu(\omega) \boldsymbol{H}$; you may use the impedance of free space, $Z_0 = \sqrt{\mu_0/\varepsilon_0}$, to simplify the equations.)

d) Find the conditions under which the reflected beam for the p-polarized incident light vanishes.

e) Find the conditions under which the reflected beam for the s-polarized incident light vanishes.

1) As an optical engineer, you are asked to design a finite conjugate microscope as shown in the figure below, using parts available to you at your company's workshop. The microscope is a combination of the objective lens and the eyepiece.



Fortunately, your company has collected over the years all kinds of parts that you can use for this, including:

- Optical tubes of any length (OTL) ranging from 170 – 200 mm
- Lenses of any diameter between 6 – 18 mm,
- and of any focal length between 5 – 50 mm.

Using these components:

a) (15 pts) Design a microscope with a visual magnification, m_V, between 50X – 100X, which is the product of the objective lens magnification and the eyepiece MP. Pick a value, select the focal length of your eye piece, and determine the working distance in object space of the microscope.

b) (15 pts) At this working distance, your microscope will have to operate with a numerical aperture, *NA*, between 0.15 – 0.25. Select the stop location such that your microscope operates telecentric in object space, select a nominal value for *NA,* and determine the required Stop diameter to produce this NA.

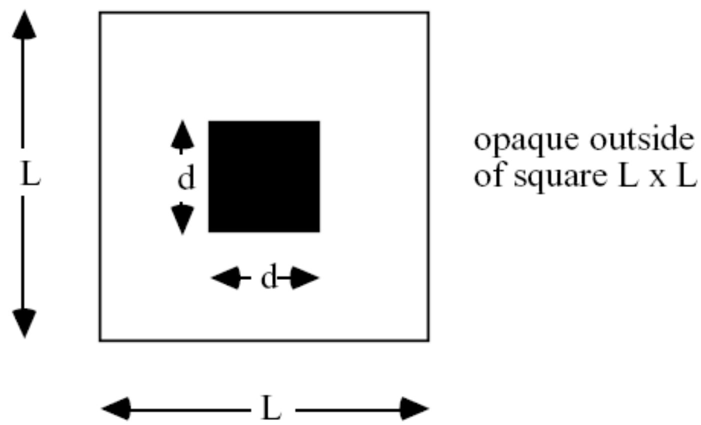
c) (10 pts) At this point, you have almost completed your microscope design! Select a lens diameter for your microscope objective and determine the maximum object size (+/- *h*) that you can image with this instrument while unvignetted. If it is not possible for the system to be unvignetted with your chosen design, explain why in terms of marginal and chief ray heights.

Assume unit-amplitude, normally incident plane-wave illumination on the aperture below.

a) (25%) Write the aperture transmission function in terms of simple rect functions using Babinet's principle.

b) (50%) Write an expression for the Fraunhoffer complex electric field distribution at a distance z_0 and wavelength λ.

c) (25%) Find the irradiance in the Fraunhoffer plane using constant $C_I = \frac{1}{2} cn\varepsilon_0$.



You may find the following Fourier Transform pairs useful:

$$\mathrm{rect}\left(\frac{x}{a}\right) \rightarrow a\,\mathrm{sinc}\left(a\xi\right)$$

$$\delta\left(x \pm x_0\right) \rightarrow e^{\pm j2\pi x_0 \xi}$$

$$e^{\pm j2\pi x \xi_o} \rightarrow \delta\left(\xi \mp \xi_0\right)$$

Consider a particle of mass m confined to a symmetric 2D harmonic potential

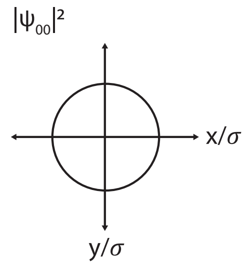
$$V(x,y) = \tfrac{1}{2}m\omega^2(x^2+y^2).$$

Energy eigenfunctions of this system can be expressed as $\psi_{mn}(x,y) = \phi_m(x)\phi_n(y)$, where $\phi_m(x)$ and $\phi_n(y)$ are the energy eigenfunctions for 1D harmonic potentials $V(x,0)$ and $V(0,y)$, respectively.

See footnotes for helpful pointers.

(a) What are the energy eigenvalues[1] of the lowest three energy eigenfunctions, ψ_{00}, ψ_{10}, ψ_{01}? [2 pt]

(b) Sketch the position-space probability densities of the first two excited states of the 2D harmonic oscillator—$|\psi_{01}(x,y)|^2$ and $|\psi_{10}(x,y)|^2$—as contours for $|\psi|^2 = 0.5\,|\psi|^2_{\text{max}}$. The ground state is plotted below as an example, with axes scaled by the ground state width $\sigma = \sqrt{h/(m\omega)}$. *Note*[2]. [2 pt]



(c) Consider the wavefunction

$$\psi_+(x,y) = \frac{1}{\sqrt{2}}\left(\psi_{10}(x,y) + i\psi_{01}(x,y)\right)$$

i. Show that ψ_+ is an energy eigenfunction of the 2D harmonic oscillator and compute its energy eigenvalue. [2 pt]

ii. Express $|\psi_+(x,y)|^2$ in terms of $\psi_{10}(x,y)$ and $\psi_{01}(x,y)$. [1 pt]

iii. Sketch $|\psi_+(x,y)|^2$ as contours in the x-y plane, as above. [1 pt]

(d) Show that ψ_+ is an eigenfunction of the 2D angular momentum operator

$$\hat{L} = \hat{x}\hat{p}_y - \hat{y}\hat{p}_x = i\hbar(\hat{a}_x^-\hat{a}_y^+ - \hat{a}_y^-\hat{a}_x^+)$$

where $\hat{p}_k$ is the linear momentum operator, $\hat{a}_k^-$ ($\hat{a}_k^+$) is the lowering (raising) operator, and $k = x, y$.[3] You may use either operator representation[4]. [2 pt]

---

[1] Note that the system Hamiltonian can be written $H = H_x + H_y$ where $H_{x,y}$ are for 1D oscillators.

[2] It may be useful to recall that $\phi_0(x) \propto e^{-x^2/2\sigma^2}$ and $\phi_1(x) \propto x\phi_0(x)$ for a 1D oscillator

[3] For example, $\hat{p}_x = -i\hbar\partial/\partial x$, $\hat{a}_x^-\phi_m(x) = \sqrt{m}\phi_{m-1}(x)$, and $\hat{a}_x^+\phi_m(x) = \sqrt{m+1}\phi_{m+1}(x)$.

[4] If using $\hat{a}_k^\pm$, note that, for example, $\hat{a}_x^-\hat{a}_y^+\phi_m(x)\phi_n(y) = (\hat{a}_x^-\phi_m(x))(\hat{a}_y^+\phi_n(y))$. If using $\hat{p}_k$, see footnotes 2-3. Note that the $\hat{a}_k^\pm$ approach involves significantly less algebra.