

**Spring 2016 Written Comprehensive Exam
Opti 501**

System of units: MKSA

In electromagnetic theory, a wave equation is a 2nd order partial differential equation that is satisfied by a single field, such as the scalar potential $\psi(\mathbf{r}, t)$, the vector potential $\mathbf{A}(\mathbf{r}, t)$, the electric field $\mathbf{E}(\mathbf{r}, t)$, or the magnetic field $\mathbf{H}(\mathbf{r}, t)$. Examples of the wave equation (in the Lorenz gauge) are

$$\nabla^2 \psi(\mathbf{r}, t) - \frac{1}{c^2} \frac{\partial^2 \psi(\mathbf{r}, t)}{\partial t^2} = -\frac{1}{\epsilon_0} \rho_{\text{total}}(\mathbf{r}, t).$$

$$\nabla^2 \mathbf{A}(\mathbf{r}, t) - \frac{1}{c^2} \frac{\partial^2 \mathbf{A}(\mathbf{r}, t)}{\partial t^2} = -\mu_0 \mathbf{J}_{\text{total}}(\mathbf{r}, t).$$

In this problem you are asked to derive a wave equation for the electric field, and another wave equation for the magnetic field, under certain special circumstances. Limiting the time-dependence of the fields to single-frequency (i.e., monochromatic) behavior, the fields will be written as $\mathbf{E}(\mathbf{r}, t) = \mathbf{E}(\mathbf{r}) \exp(-i\omega t)$ and $\mathbf{H}(\mathbf{r}, t) = \mathbf{H}(\mathbf{r}) \exp(-i\omega t)$. Consequently, the desired wave equations will be 2nd order partial differential equations in spatial coordinates only—one satisfied by $\mathbf{E}(\mathbf{r})$, the other by $\mathbf{H}(\mathbf{r})$. It will be further assumed that free charge and free current are absent from the system, that is, $\rho_{\text{free}}(\mathbf{r}, t) = 0$ and $\mathbf{J}_{\text{free}}(\mathbf{r}, t) = 0$, and that the electromagnetic fields reside in a homogeneous, linear, isotropic medium specified by its permittivity $\epsilon_0 \epsilon(\omega)$ and permeability $\mu_0 \mu(\omega)$.

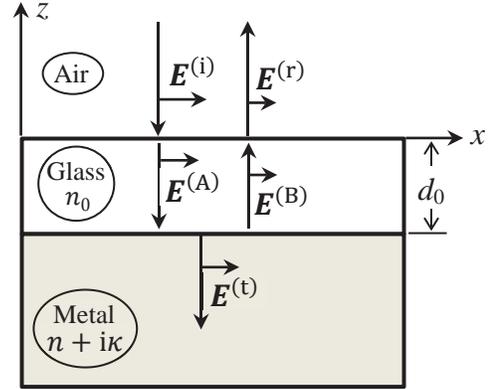
- 4 Pts a) Write Maxwell's equations for the \mathbf{E} and \mathbf{H} fields, given the above restrictions.
- 4 Pts b) Eliminate the \mathbf{H} field from Maxwell's equations in order to arrive at the wave equation satisfied by $\mathbf{E}(\mathbf{r})$.
- 2 Pts c) Eliminate the \mathbf{E} field from Maxwell's equations in order to arrive at the wave equation satisfied by $\mathbf{H}(\mathbf{r})$.

Hint: The identity $\nabla \times [\nabla \times \mathbf{F}(\mathbf{r})] = \nabla[\nabla \cdot \mathbf{F}(\mathbf{r})] - \nabla^2 \mathbf{F}(\mathbf{r})$ is valid for any vector field $\mathbf{F}(\mathbf{r})$ residing in 3-dimensional space.

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A smooth and flat metallic surface is coated with a thin dielectric layer of thickness d_0 and real-valued refractive index n_0 . The complex refractive index of the metal is $\sqrt{\mu(\omega)\epsilon(\omega)} = n + i\kappa$, where both n and κ are real-valued and positive. A linearly-polarized homogeneous plane-wave of frequency ω and vacuum wavelength $\lambda_0 = 2\pi c/\omega$ is normally incident from the air onto the coated metallic surface.

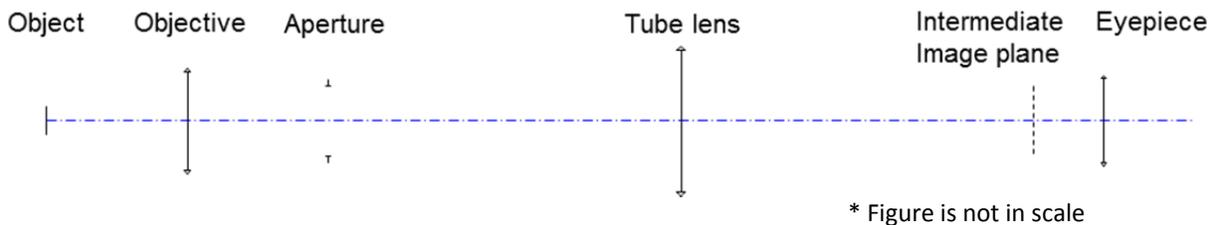


- 3 Pts a) Write expressions for the E - and H -fields in the medium of incidence (air), in the dielectric layer, and in the semi-infinite metallic substrate. Assume $\mu(\omega) = 1$ at the optical frequency ω , the speed of light in vacuum $c = 1/\sqrt{\mu_0\epsilon_0}$, and the impedance of free space $Z_0 = \sqrt{\mu_0/\epsilon_0}$.
- 3 Pts b) Match the boundary conditions at the top and bottom surfaces of the dielectric layer (located at $z = 0$ and $z = -d_0$) in order to arrive at relations among the various unknown parameters.
- 3 Pts c) Express the Fresnel reflection coefficient $\rho = E^{(r)}/E^{(i)}$ in terms of n , κ , n_0 , d_0 , ω , and c . The final result will be easier to analyze when expressed as a function of $\rho_1 = (1 - n_0)/(1 + n_0)$, $\rho_2 = [n_0 - (n + i\kappa)]/[n_0 + (n + i\kappa)]$, and the round-trip phase $\varphi_0 = 4\pi n_0 d_0/\lambda_0$.
- 1 Pts d) Assuming that the thickness d_0 of the dielectric layer is adjustable, when will the reflectance $R = |\rho|^2$ of the coated metallic surface reach a minimum or a maximum? What are the values of R_{\min} and R_{\max} ?
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Spring 2016 Written Comprehensive Exam

OPTI502

A compound microscope consists of an infinite conjugate objective, a tube lens with a focal length of 200 mm, and a 10x eyepiece. The objective has a numerical aperture (NA) of 0.25, and a focal length of 20 mm. The maximum object size is ± 1 mm. The aperture stop of the system is located so that the system is telecentric in object space. The eyepiece also must observe the intermediate image under the telecentric condition. Consider all of the lenses to be thin lenses in air.



For parts 1-4, the object is in air.

1. Sketch the chief ray and the marginal ray. (1 pt)
2. Determine the required separation between the objective and the tube lens? (1 pt)
3. Calculate the following parameters:
 - a. the focal length of the eyepiece, the separation between the objective and the aperture stop, and the separation between the tube lens and the intermediate image plane. (1 pt)
 - b. the diameter aperture stop. (1 pt)
 - c. the diameter of each lens required for the system to operate without vignetting. (2 pts)
4. What is the total magnification of this microscope? (2 pts)
5. This microscope is now to be used to image an object located 5 mm below a flat water surface (refractive index $n=1.33$). What is the required distance between the object and objective to obtain an in-focus image? Assume no aberrations will be introduced by the water and consider only the axial object point. (2 pts)

Spring 2016 Written Comprehensive Exam

OPTI 502

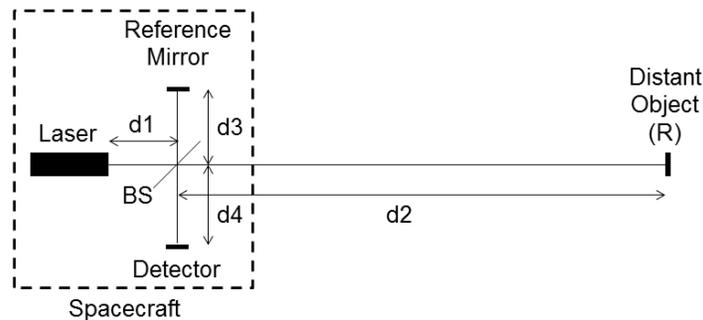
An $F/\# = 2.2$ Retrofocus lens consists of two separated thin lenses. The first lens has a focal length of -100 mm and the second lens has a focal length of 26 mm. The thin lenses are separated by a distance of 20 mm and the aperture stop is located halfway between the two lenses. The lens is used with a full-frame sensor with dimensions 36 mm x 24 mm in the horizontal and vertical directions, respectively.

1. What is the Back Focal Distance (BFD)? (2 pts)
2. What is the effective focal length f_E ? (2 pts)
3. Where is the rear principal plane P' located relative to the last surface? (2 pts)
4. What is the entrance pupil diameter (EPD) of the lens? (2 pts)
5. What are the full fields of view in the horizontal and vertical directions? (2 pts)

Spring 2016 Written Comprehensive Exam

OPTI 505R

A Twyman-green interferometer is used on a spacecraft to detect other objects in space, as shown below. The test arm of the interferometer is a telescope that transmits and receives light reflected from the other objects. Laser wavelength is $1\mu\text{m}$. The beamsplitter is a perfect 50% transmitting/reflecting surface, irradiance at the output of the laser is I_0 , and the reference mirror is 100% reflecting.



1.) (2 pts) Write an expression for the optical path length (OPL) of the test arm in terms of the d-distances shown on the diagram. Assume a vacuum medium.

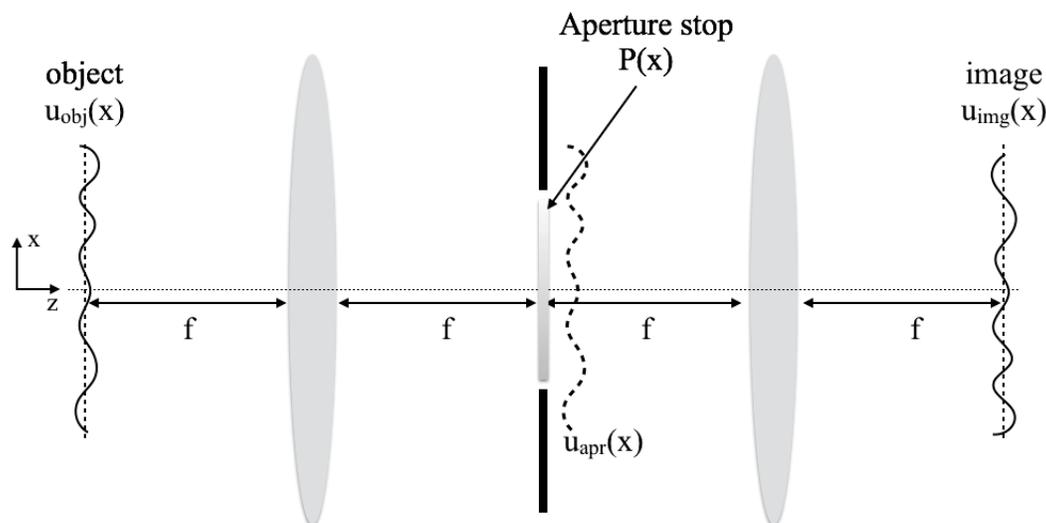
2.) (2 pts) Write an expression for the OPD between the test arm and the reference arm in terms of the d-distances shown on the diagram. Assume a vacuum medium.

3.) (2 pts) Write an expression for the irradiance at the detector as a function of the OPD in (2), assuming that the light returned from the distant object can be modeled as a flat mirror reflection with $R \approx 0.0001$.

4.) (2 pts) Compare the signal level received by the spacecraft from just the light reflected from the distant object, which is proportional to R , to the interferometer signal based on (3) from the same reflection through the full range of one wavelength in OPD. That is, the interferometer signal is the "fringe" term irradiance variation.

5.) (2 pts) What is the maximum distance d_2 that the distant object can be from the spacecraft when the bandwidth of the laser is 100 Hz. Hint: Use your knowledge of temporal coherence to set a maximum OPD before unacceptable fringe visibility.

Consider the following coherent 4-f imaging system with an aperture stop, with transmittance $P(x)$, placed between two identical lenses:



The object and image fields are specified by $u_{obj}(x)$ and $u_{img}(x)$ respectively and the object field is quasi-monochromatic at $\lambda = 500nm$. The front and back focal lengths of the two identical lenses are $f = 10cm$. In this system you may assume each lens is ideal, that is the lens has no aberrations and it is of infinite extent, so you can ignore any diffraction from the lens itself.

(a) Given a sinusoidal object field: $u_{obj}(x) = 1 + \cos(2\pi\xi_o x)$, derive the field $u_{apr}(x)$ in the aperture stop plane (located at $2f$) in the imaging system and compute the location of the optical spots for the spatial frequency $\xi_o = 20cycles/mm$.

[2 points]

(b) Given a clear aperture stop of size $D = 1cm$, $P(x) = \text{rect}\left(\frac{x}{D}\right)$, what is the highest spatial frequency (ξ_{max}) in the object field that will pass through this imaging system?

[2 points]

(c) For the aperture stop defined in (b), compute the image field $u_{img}(x)$ for the object field: $u_{obj}(x) = 1 + \cos^2(2\pi\xi_a x) + \cos(2\pi\xi_b x)$, where $\xi_a = 60cycles/mm$ and $\xi_b = 40cycles/mm$.

[2 points]

(d) Now consider the aperture stop defined in (b), where a phase-mask is inserted in the lower part of the stop such that: $P(x) = \text{rect}\left(\frac{x}{D}\right) \cdot e^{+j\pi\phi(x)}$ and $\phi(x) = \text{rect}\left(\frac{x+D/4}{D/2}\right)$.

Compute the image field $u_{img}(x)$ for the object field $u_{obj}(x) = \cos(2\pi\xi_o x)$. *Sketch and comment on the relation between the object and the image irradiance.*

[4 points]

Spring 2016 Comprehensive Exam
OPTI 507

(a)

Draw a schematic diagram of spontaneous light-phonon scattering for both Stokes and anti-Stokes processes. Use solid arrows for the light vector and wiggly line arrows for the phonon vector, and indicate the scattering angle θ . Write down both energy and momentum conservation for both Stokes and anti-Stokes processes. It is not necessary and requested for you to assume any particular form of dispersion (frequency-dependence of wave vector). What is the difference between Brillouin and Raman scattering?

(6 points)

(b)

Consider coherent anti-Stokes Raman spectroscopy (CARS) as an example for coherent light scattering. Draw the energy diagram for CARS and write down the equation for energy conservation. Does this process involve acoustic or optical phonons? Assume the relevant phonon frequency to be $\hbar\Omega = 25\text{meV}$ and that you have two laser sources with frequencies ω_1 and ω_2 , respectively. For $\omega_1 > \omega_2$ and $\hbar\omega_1 = 1.2\text{eV}$, determine the frequency (in units of eV) of the CARS signal.

(4 points)

Spring 2016 Comprehensive Exam
OPTI 507

Consider a simple monoclinic lattice with $\vec{a} = (a \sin \beta, 0, a \cos \beta)$, $\vec{b} = (0, b, 0)$, and $\vec{c} = (0, 0, c)$. For $a = 0.5\text{nm}$, $b = 0.7\text{nm}$, $c = 0.4\text{nm}$ and $\beta = 50^\circ$, determine the volume of the unit cell. Also, derive expressions (as functions of a, b, c, β) for the three primitive translation vectors in reciprocal space.

(10 points)

Consider a hydrogen atom at rest in the state $|200\rangle$, where the integers in the ket indicate the usual principle and orbital angular momentum quantum numbers n , l , and m_l respectively, and m_l is the quantum number associated with electron orbital angular momentum about the z axis. In this problem, we neglect spin angular momentum of the electron and the proton, so that the only atomic angular momentum that we need to consider is the electron's orbital angular momentum about the nucleus.

(a – 7 points) Describe an experimentally realistic procedure that will put the atom into a superposition state for which there is a 50% probability of finding the atom in state $|200\rangle$, and a 50% probability of finding the atom in state $|410\rangle$. Specifically, determine how to accomplish this task using a pulse of light from a monochromatic laser beam that has an electric field amplitude of $\mathcal{E}_0 = 10$ V/m, and neglect spontaneous emission effects. Specify the wavelength of the laser light desired, the duration of the laser pulse (assuming a constant laser power during the pulse), the polarization direction and propagation direction of the laser light (be as specific as you can), and any other parameters that you think are important. Note that there are multiple answers that may be considered correct, but your answer must be physically realistic. If you do not remember some of the quantitative expressions that are needed for this problem, for partial credit describe qualitatively what must be considered in as much detail as you can, and specify and describe any relevant approximations involved in the calculations.

(b – 3 points) If the hydrogen atom is known to be in state $|410\rangle$ at some time t , list all of the possible states into which it can then decay by the spontaneous emission of a single photon.

Helpful expressions:

Ionization energy of hydrogen in the ground state: 13.6 eV

$\hbar \approx 1.1 \times 10^{-34}$ J·s $\approx 6.6 \times 10^{-16}$ eV·s

$a_0 \approx 5.3 \times 10^{-11}$ m (Bohr radius)

$e \approx 1.6 \times 10^{-19}$ C (quantum of electric charge)

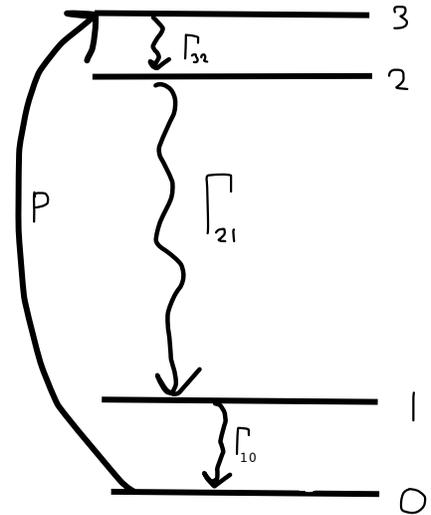
Cartesian components of angular matrix element

$$\langle l = 1, m_l = 0 | \hat{\mathbf{r}} | l = 0, m_l = 0 \rangle = (0, 0, \sqrt{\frac{1}{3}})$$

Radial matrix elements for atomic hydrogen

$$\langle n = 2, l = 0 | r | n = 4, l = 1 \rangle = 1.28a_0$$

Consider a laser composed of a stable linear cavity, with 2 mirrors each having power reflectivity coefficients $R_1 = 0.99$ and $R_2 = 0.95$, and a homogeneously broadened 15 cm long gain medium. The gain medium is a multi-level system as shown in the figure, with population densities N_0, N_1, N_2, N_3 . The lasing transition is between levels 2 and 1. The total population density of this closed system is $N_T = 1 \times 10^{19} \text{cm}^{-3}$. The population transition rates (Γ_{ij}) are also shown, where Γ_{21} is due entirely to spontaneous emission. An external pumping rate P is required for population inversion. The maximum gain cross section for the $2 \rightarrow 1$ atomic transition is $\sigma(\nu_o) = 3 \times 10^{-21} \text{cm}^2$. **In this problem, assume that Γ_{32} is infinite (fast decay).**



- (a) Write down the relevant population rate equations in the small-signal limit (i.e. do not include stimulated transitions). **(2 pts)**
- (b) Using results from part (a), solve for N_2/N_1 in the steady-state, expressed in terms of P, Γ_{21} , and Γ_{10} . **(2 pts)**
- (c) Under what condition can a steady-state population inversion be obtained? **(1 pts)**
- (d) Calculate the maximum (on resonance) small-signal gain coefficient assuming $P = \Gamma_{10} = 2 \times \Gamma_{21}$. **(3 pts)**
- (e) Calculate the threshold gain coefficient. Using your result from part (d) above, determine if lasing action is possible. Justify your answer by clearly showing your work. **(2 pt)**

Spring 2016 Written Comprehensive Examination
OPTI-537

Answer the following questions related to solid-state physics and semiconductor detectors. All parts weighted as indicated.

(a) (1 Point) Write the 3D time-independent Schrödinger equation for a single electron in a potential with periodicity defined by a Bravais lattice. Show where the periodicity is expressed in the Hamiltonian and explain what assumptions are necessary to invoke the time-independent form of the Schrödinger equation to describe the electronic structure and related properties of crystals.

(b) (1 Point) Write the expression that defines the relationship between the direct (\mathbf{R}) and reciprocal (\mathbf{G}) lattice vectors for a perfect crystal. Define all terms.

(c) (1 Point) Explain what is meant by the 1st Brillouin zone, and write an expression for its volume for an orthorhombic lattice with primitive direct lattice constants $a=3 \text{ \AA}$, $b=4 \text{ \AA}$, and $c=5 \text{ \AA}$.

(d) (1 Point) If a macroscopic crystal of the lattice structures in (c) above has dimensions of $3\text{mm} \times 2\text{mm} \times 5\text{mm}$ in the $\hat{\mathbf{x}} = \hat{\mathbf{a}}$, $\hat{\mathbf{y}} = \hat{\mathbf{b}}$, and $\hat{\mathbf{z}} = \hat{\mathbf{c}}$ directions respectively, and we invoke Bloch's theorem and Born-Von Karman boundary conditions, what is the spacing between the allowed k_x , k_y , and k_z states? (Express your answer in the customary units of cm^{-1}). How many electrons fit inside a single band in the 1st Brillouin zone of this crystal?

(e) (2 Points) Explain the concept known as the Ewald sphere and its utility for explaining the elastic scattering (diffraction) of electrons and light from a crystal.

Then consider a 2D rectangular lattice with primitive lattice constants $a=3 \text{ \AA}$ and $b=4 \text{ \AA}$. Use the Ewald construction to work out the allowed reflections (\mathbf{k}_{out}) if say light has an incident \mathbf{k} vector (in units of \AA^{-1})

$$\mathbf{k}_{\text{inc}} = \frac{2\pi}{6} \hat{\mathbf{A}} + \frac{2\pi}{4} \hat{\mathbf{B}},$$

where $\hat{\mathbf{A}}$ and $\hat{\mathbf{B}}$ are the unit vectors in reciprocal space corresponding to directions $\hat{\mathbf{a}}$ and $\hat{\mathbf{b}}$ in real space.

(f) (1 Point) What is the wavelength of the light in problem (e) above (in Angstroms?) Approximately what photon energy is this in eV?

(g) (1 Point) Make a sketch that shows the concentrations of majority and minority charge carriers versus temperature ranging from 0K to 500K for a typical crystal of N-type Si semiconductor. Annotate the plot with an explanation of what physical effects are being revealed.

(h) (2 Points) Draw the band structure of an unbiased PN junction. Sketch the locations of donor and acceptor dopant states relative to the conduction and valence bands, and indicate where the Fermi level is at room temperature. Label which side is P and which is N and clearly indicate majority and minority carriers. Finally, clearly diagram and label the 2 electron and 2 hole currents present that exactly balance when the diode is in thermal equilibrium.

Answer the following questions related to radiative transport. The radiative transport equation (RTE), or Boltzmann equation, describes the time evolution of a phase-space distribution function w in terms of four processes: absorption, emission, propagation, and scatter

$$\frac{dw}{dt} = \left[\frac{\partial w}{\partial t} \right]_{abs} + \left[\frac{\partial w}{\partial t} \right]_{emiss} + \left[\frac{\partial w}{\partial t} \right]_{prop} + \left[\frac{\partial w}{\partial t} \right]_{scat} \quad (1)$$

(a) (1 point) What is w a function of and what are its units? What does it describe?

(b) (2 points) In class, we derived forms for each of the terms in the RTE and wrote an overall spatio-temporal-integro-differential transport equation of the form:

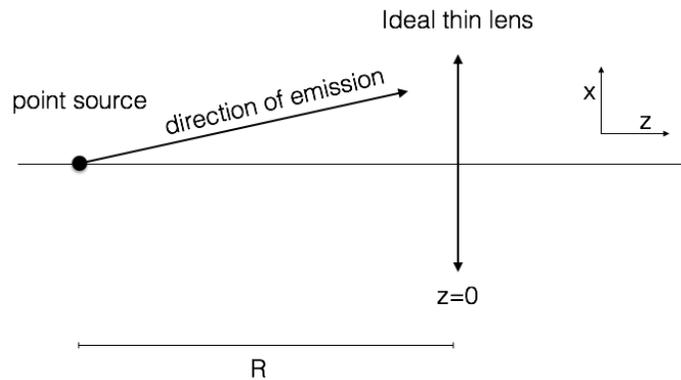
$$\frac{dw}{dt} = -c_m \mu_{total} w + \Xi_{p,E} - c_m \hat{s} \cdot \nabla w + \mathbf{K}w \quad (2)$$

where \mathbf{K} is an integral operator. Write a general expression for the \mathbf{K} operator for both inelastic collisions and elastic collisions.

(c) (2 points) What are the limitations of the RTE? In other words, are there optical processes that cannot be modeled with the RTE?

(d) (5 points) For the rest of the questions, please consider the propagation of photons is limited to one plane (problem is in 2 dimensions and not 3).

Consider a source emitting incoherent light at a constant rate but only in **one** direction towards a thin lens. Assume that scattering in air **cannot** be neglected but that absorption can. Further assume that when a scatter occurs, it is elastic and the angle of scatter is isotropic in all directions. A distance R away from the point source is an ideal thin lens of infinite extent (see picture). Assume that the direction of emission from the source is towards this thin lens. Write (but do not solve) an RTE that fully describes this system. If you introduce terms, please describe what they are.



(HINT 1) It may be helpful to remember that the ABCD matrix for an ideal thin lens is $\begin{bmatrix} 1 & 0 \\ -1/f & 1 \end{bmatrix}$.

(HINT 2) An ideal thin lens changes the direction of a ray. The only term in the RTE that changes ray direction is the scattering kernel.

Consider a hydrogen atom at rest in the state $|200\rangle$, where the integers in the ket indicate the usual principle and orbital angular momentum quantum numbers n , l , and m_l respectively, and m_l is the quantum number associated with electron orbital angular momentum about the z axis. In this problem, we neglect spin angular momentum of the electron and the proton, so that the only atomic angular momentum that we need to consider is the electron's orbital angular momentum about the nucleus.

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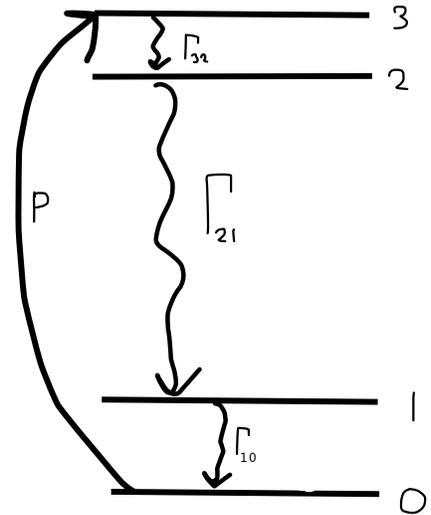
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- (b) Using results from part (a), solve for N_2/N_1 in the steady-state, expressed in terms of P, Γ_{21} , and Γ_{10} . **(2 pts)**
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- (d) Calculate the maximum (on resonance) small-signal gain coefficient assuming $P = \Gamma_{10} = 2 \times \Gamma_{21}$. **(3 pts)**
- (e) Calculate the threshold gain coefficient. Using your result from part (d) above, determine if lasing action is possible. Justify your answer by clearly showing your work. **(2 pt)**

Spring 2016 Written Comprehensive Exam
Opti 546

Consider the situation where an initial field $E(x', y', 0)$ at $z = 0$ propagates in vacuum along the z -axis. Then for $z > 0$ the field may be expressed as the following diffraction integral based on the Huygens-Fresnel principle

$$E(x, y, z) = \frac{1}{i\lambda} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} dx' dy' E(x', y', 0) \left(\frac{e^{ikr}}{r} \right) \left(\frac{z}{r} \right), \quad (1)$$

where $k = 2\pi/\lambda$, and $r = \sqrt{(x - x')^2 + (y - y')^2 + z^2}$.

(a - 3pts) List the key ideas and approximations that underly the reduction of the above diffraction integral to its form in the Fresnel approximation

$$E(x, y, z) \approx \frac{e^{ikz}}{i\lambda z} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} dx' dy' E(x', y', 0) e^{\frac{ik[(x-x')^2 + (y-y')^2]}{2z}}. \quad (2)$$

(b - 2pts) Building upon part (a) list the approximations that underly the reduction of the Fresnel approximation in Eq. (2) to its form in the Fraunhofer region

$$E(x, y, z) \approx \frac{e^{ikz}}{i\lambda z} e^{\frac{ik(x^2 + y^2)}{2z}} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} dx' dy' E(x', y', 0) e^{-\frac{ik(xx' + yy')}{z}}. \quad (3)$$

(c - 2pts) For an initial field at $z = 0$ we hereafter use the example of an incident plane-wave that is truncated by a circular aperture of radius 'a' that is centered on the origin. Then based on your analysis from part (b), show that the Fraunhofer region arises for $z \gg z_0$ where the Rayleigh range is given by $z_0 = ka^2/2$.

(d - 2pts) Sketch the variation of the on-axis intensity $|E(0, 0, z)|^2$ versus propagation z distance past the uniformly illuminated circular aperture, assuming $|E(0, 0, 0)|^2 = 1$. You should indicate the propagation regions corresponding to the Fresnel and Fraunhofer regions with reference to your results from part (c).

(e - 1pt) *Explain* what is meant by the term 'Airy disk' as applied to the present problem in the Fraunhofer region?

Spring 2016 Written Comprehensive Exam Opti 546

This problem deals with scattering of a plane-wave of wavelength λ from an acousto-optic cell of length L with spatially periodic refractive-index $\Delta n(x) = n_1 \sin(Kx)$, where n_1 is the magnitude of the index modulation, and $K \ll k$ the magnitude of the wavevector of the acoustic wave, k being the magnitude of the optical wavevector.

(a - 2pts) Sketch the characteristic arrangement of the incident optical field wavevector, acoustic wavevector, and the scattered field components for Bragg diffraction for the situation described above.

(b - 3pts) Considering now Raman-Nath diffraction the field exiting the acousto-optic cell is in the quasi-static approximation

$$\mathcal{E}(x, L) = \mathcal{E}_i e^{i2\pi\Delta n(x)L/\lambda}, \quad (1)$$

where \mathcal{E}_i is the amplitude of the incident plane-wave. Using the Bessel function identity

$$e^{i\delta \sin \phi} = \sum_{m=-\infty}^{\infty} J_m(\delta) e^{im\phi}, \quad (2)$$

with $J_m(\delta)$ the Bessel function of the first kind of order m , show that the field exiting the acousto-optic cell is composed of scattered waves with components of their wavevectors along the x-axis given by $k_x^{(m)} = mK, m = 0, \pm 1, \pm 2, \dots$

(c - 2pts) Based on your solution from part (b) argue that in the far field region beyond the acousto-optic cell the intensity pattern will be composed of scattered waves traveling at angles $\theta_m = mK/k$ with respect to the z-axis, and with intensities $I_m(L) = I_i J_m^2(2\pi n_1 L/\lambda)$, with I_i the incident intensity.

(d - 1pt) Briefly discuss the meaning of the quasi-static approximation alluded to above.

(e - 2pts) Discuss whether the various scattering orders incur any frequency shifts as a consequence of the acousto-optic interaction given a sound frequency Ω .