Fall 2015 Written Comprehensive Exam
Opti 501

System of units: MKSA

2 Pts a) The charge-current continuity equation is written

$$\boldsymbol{\nabla}\cdot\boldsymbol{J}(\boldsymbol{r},t)+\partial\rho(\boldsymbol{r},t)/\partial t=0.$$

Explain in a few sentences the physical meaning of the equation and also the meaning of the various terms and symbols that appear in the equation. Specify the units of $\boldsymbol{J}$ and ρ.

3 Pts b) Derive the charge-current continuity equation for free charge-density $\rho_{\text{free}}(\boldsymbol{r},t)$ and free current-density $\boldsymbol{J}_{\text{free}}(\boldsymbol{r},t)$ from Maxwell's *microscopic* equations.

Note: The microscopic equations of Maxwell do *not* contain polarization and magnetization.

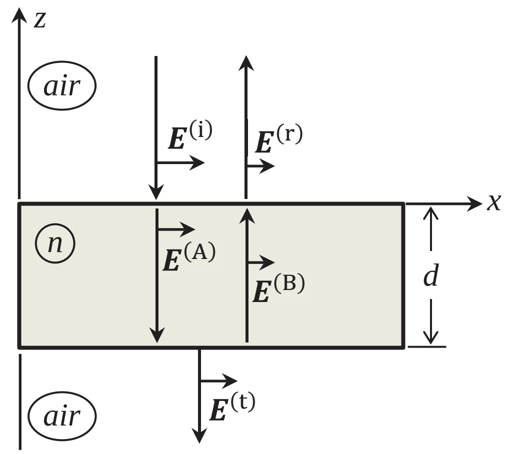
2 Pts c) Write down the definitions of bound electric charge-density and bound electric current-density, then relate them to each other via the corresponding continuity equation.

3 Pts d) Derive the charge-current continuity equation for bound electric charge and bound electric current from Maxwell's *macroscopic* equations.

Note: You may now set $\rho_{\text{free}}=0$ and $\boldsymbol{J}_{\text{free}}=0$ in Maxwell's macroscopic equations.

Fall 2015 Written Comprehensive Exam
Opti 501

<u>System of units: MKSA</u>



A monochromatic, linearly-polarized, homogeneous plane-wave is normally incident on a slab of transparent dielectric material of thickness d and (real-valued) refractive index n, as shown.

3 Pts a) Write expressions for the E- and H-fields in the medium of incidence (air), in the dielectric slab, and in the region below the slab, in which the transmitted beam emerges.

(You may assume $\mu = 1$, incident angular frequency $= \omega$, speed of light in vacuum $c = 1/\sqrt{\mu_0 \varepsilon_0}$, and impedance of free space $Z_0 = \sqrt{\mu_0/\varepsilon_0}$.)

2 Pts b) Match the boundary conditions at the top surface $(z = 0)$ and at the bottom surface $(z = -d)$ of the slab in order to arrive at relations among the various unknown parameters.

2 Pts c) Find expressions for the Fresnel reflection and transmission coefficients $\rho = E^{(\mathrm{r})}/E^{(\mathrm{i})}$ and $\tau = E^{(\mathrm{t})}/E^{(\mathrm{i})}$ in terms of n, d, ω, and c. (**Note**: In air, the incident wavelength is $\lambda_0 = 2\pi c/\omega$.)

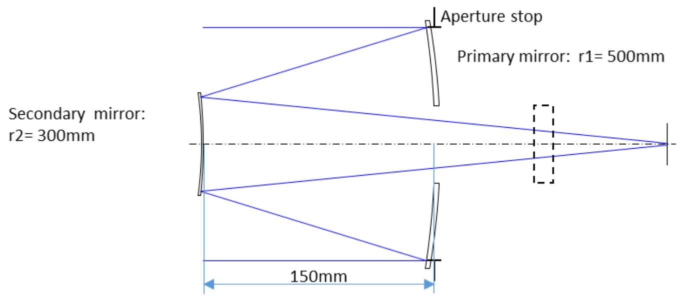
2 Pts d) Under what circumstances will the reflectance $R = |\rho|^2$ of the slab be at a minimum? What is the value of $R_{\min}$?

1 Pt e) When will the reflectance of the slab be at a maximum, and what is the value of $R_{\max}$?

Fall 2015 Written Comprehensive Exam

OPTI502

A Cassegrain telescope consists of a concave primary mirror followed by a convex secondary mirror. The figure below shows the optical layout with parameters without conic constants.



The aperture stop is located at the primary mirror with a diameter of 150 mm. The field of view of the telescope is +/- 1.0 degrees. The objects are at infinity.

1. What are the focal length of this telescope and the image space F/#? (1 point)
2. Where are the principal planes of the system? (2 points)
3. Where is the paraxial image plane relative to the secondary mirror? (1 point)
4. Trace the chief ray and determine chief ray height on the secondary mirror and the maximum image height. (2 point)
5. What are the sizes and locations of the entrance pupil and exit pupil? (2 points)
6. Where should the aperture stop be moved to achieve image space telecentricity? (1 point)
7. If a BK7 glass window (n=1.517) with 20mm thickness is placed in the image space, where is the paraxial image plane? (1 point)

Fall 2015 Written Comprehensive Exam

OPTI 502

	O		1		2		3		O'
f			10.18914		-5.43647		12.4478		
−ϕ			-0.09814		0.183943		-0.08034		
t				1.01346		2.77622		$\boxed{t_3'}$	
y(marginal)			3.664487		3.3		3.986742		0
u(marginal)		$\boxed{u_1}$		-0.35965		0.247366		-0.07291	
y(chief)			-0.4777		0		$\boxed{\bar{y}_3}$		21.33398
u(chief)		0.424475		0.471358		0.471358		0.366232	

The spreadsheet above shows a paraxial raytrace for a Cooke triplet with thin lens elements of focal lengths f_1 = 10.18914 mm, f_2 = -5.43647 mm and f_3 = 12.4478 mm, respectively. The spacing between the elements are $t_1' = 1.01346\text{mm}$ and $t_2' = 2.77622\text{mm}$, respectively. The object O is at infinity and the image O' is formed a distance t_3' after the last element. The paraxial marginal and chief rays have been traced.

(a) Several of the raytracing values are missing. What at the values of u_1, $\bar{y}_3$ and t_3'? (3 Points)

(b) Where is the aperture stop located? (1 Point)

(c) Where is the exit pupil located and what is its diameter? (2 Points)

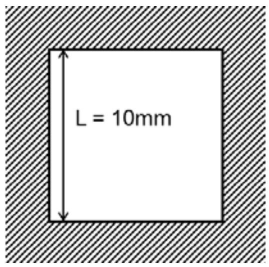
(d) What is the full field of view of the system? (1 Point)

(e) Where is the rear principal plane located? (1 Point)

(f) What is the overall power of the triplet? (1 Point)

(g) What is the value of the Lagrange Invariant for the system? (1 Point)

The square aperture shown below is illuminated by an on-axis plane wave with λ = 488nm and amplitude A V/m. Inside the square, transmission is unity. Outside of the square is opaque. The material on both sides of the aperture is air.

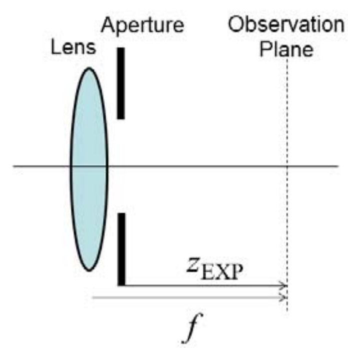


Constant
c, speed of light in vacuum
ε_0, permittivity of free space
μ_0, permeability of free space
e^-, electronic charge
h, Plank's constant
k, Boltzman's constant
N_A, Avogadro's number
m_e, electron mass
m_p, proton mass
m_n, neutron mass

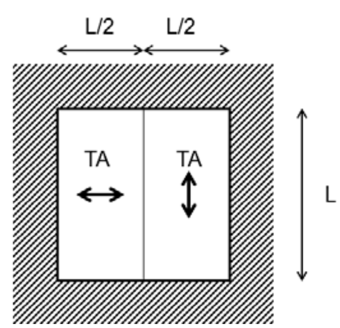
a.) (2pts) Approximately how far away from the aperture does the Fraunhofer region start? Describe your reasoning and show equations, if any.

b.) (2pts) Write an expression for the two-dimensional irradiance at a Fraunhofer plane with the proper unit conversion from electric field in V/m to irradiance in W/m^2. Keep constants and variables in their symbolic form. For example, use λ, π, etc. where appropriate. Use z_0 for the propagation distance and L for the aperture dimension. Other symbols that you may find useful are given in the table.

c.) (2pts) The aperture is used in the exit pupil of an optical system, as shown on the right, where a monochromatic point source at infinity in object space is imaged at the observation plane. Write an expression for the two-dimensional irradiance in the observation plane with the proper unit conversion from electric field in V/m to irradiance in W/m^2. Like in (b), keep constants and variables in their symbolic form. You may assume that the electric field amplitude illuminating the aperture is A V/m.

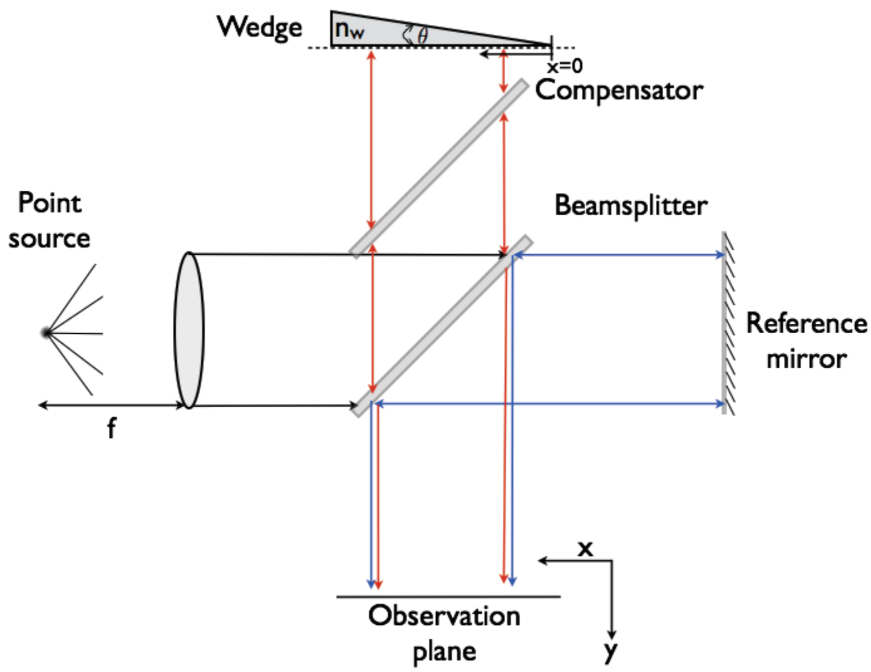


d.) (4pts) Two polarizers are placed in the aperture as shown at the right. Their transmission axes (TAs) are indicated by thick double arrows. Repeat (c) with the modified aperture. You may assume that the polarizers are perfect, so that they do not transmit any polarization components perpendicular to TA. (Hint: Assume that the point source is unpolarized.)



Note: You may use the small angle approximation as justified.

Consider the following Twyman-Green interferometer setup for measuring a test optic in one of the arm and an optical flat in the reference arm. For a monochromatic point source



illumination one can observe fringes in the observation plane, where the fringe pattern is a function of the test optic surface. Let us consider a glass wedge as the test optic, with a wedge angle of θ and refractive index n_w. Assume that the bottom surface (facing the beamsplitter) of the wedge is AR coated and reflection from that surface is negligible. You may also assume that the wedge bottom surface is parallel to the reference mirror surface in the unfolded path.

(a) What interference pattern would be observed in the observation plane (x-direction)? Express mathematically.

[2 points]

(b) What is the fringe period (along x-axis) as a function of wedge angle θ and refractive index n_w and illumination wavelength λ?

[2 points]

(c) Find the wedge angle θ if the fringe period is $\Lambda_x = 2\mu m$, given $\lambda = 500nm$ and $n_w = 1.5$

[2 points]

(d) For a quasi-monochromatic source with a rectangular power spectrum profile of spectral width $\Delta\lambda = 10nm$, sketch the fringe visibility (along x-axis) in the observation plane. Assume a center wavelength $\lambda_o = 550nm$, $n_w = 1.5$ and $\theta = 5\,\text{deg}$ and the glass beamsplitter dispersion effects are negligible over the source spectral width.

[4 points]

Fall 2015 Comprehensive Exam
OPTI 507

Within the Lorentz oscillator model, the dielectric function reads (for simplicity, we neglect damping in this problem)

$$\varepsilon(\omega) = \varepsilon_\infty - \frac{\omega_{pl}^2}{\omega^2 - \omega_0^2}$$

A slight modification of this dielectric function, sometime used as a very simply model for an optical gain medium, is given by the following dielectric function

$$\varepsilon(\omega) = \varepsilon_\infty + \frac{\omega_{pl}^2}{\omega^2 - \omega_0^2}$$

For this modified dielectric function, determine the numerical values of ω_T and $\omega_L - \omega_T$. Also, plot $\varepsilon(\omega)$, carefully labeling all special points and asympotics and making sure one can see whether ε_0 is larger than ε_∞ (your plot does not have to be to scale!), determine the width of the stop band (or reststrahlen band) in units of s^{-1} and indicate it in your plot, and indicate in your plot the regions of normal and anomalous dispersion . Assume $\varepsilon_\infty = 12$, $\omega_{pl} = 2\times10^{12} s^{-1}$, $\omega_0 = 5\times10^{15} s^{-1}$.

(10 points)

Fall 2015 Comprehensive Exam
OPTI 507

Assume you have a direct-gap semiconductor with a first-class dipole allowed transition and an electron effective mass of $m_e = 0.8 m_0$ and hole effective mass of $m_h = 1.4 m_0$ where m_0 is the electron mass in vacuum.

Furthermore, assume you have measured the energies of the 2s and 3s exciton to be

$$E_{2s} = 1.611\text{eV}$$
$$E_{3s} = 1.612\text{eV}$$

Determine the exciton Bohr radius and the band gap of the semiconductor.

Hint, you can use the following relation for the exciton binding energy:

$$E_B = \frac{\hbar^2}{2 m_r a_B^2}$$

(You may use $\hbar^2 / m_0 = 7.62$ eV Å^2.)

(10 points)

Fall 2015 Written Comprehensive Exam
OPTI 511

In this problem we consider the electromagnetic field of a single mode of an optical cavity. The field has an optical frequency ω. The state of the field is described in terms of photon number states $|n\rangle$, where n represents the number of photons in state $|n\rangle$. These states are eigenstates of the photon number operator $\hat{N}$ such that $\hat{N}|n\rangle = n|n\rangle$, and therefore they are also eigenstates of the field Hamiltonian $\hat{H} = \hbar\omega\hat{N} + \hbar\omega/2$. Below, you will work with $\hat{N}$ as well as the photon annihilation operator $\hat{a}$. Recall that $\hat{a}|n\rangle = \sqrt{n}\,|\,n-1\rangle$ for $n > 0$, and $\hat{a}|0\rangle = 0$.

For the following questions, you will use a superposition of photon number states that is defined as

$$|\psi\rangle = \frac{1}{2}|0\rangle + c|2\rangle + \frac{i}{2}|4\rangle,$$

where c is a scalar.

(a – 1.5 pt) Specify a number for c so that $|\psi\rangle$ is properly normalized. You will use this number for c in the remainder of this problem.

(b – 1 pt) Is $|\psi\rangle$ an energy eigenstate of the field? Briefly justify your answer.

(c – 1 pt) Suppose you could measure the number of photons in the field. Specify the probabilities of exactly measuring 0, 1, 2, 3, and 4 photons.

(d – 1 pt) Calculate $\langle\hat{N}\rangle$, the expectation value for the number of photons in the field.

(e – 1 pt) Calculate $\langle\hat{N}^2\rangle$.

(f – 1.5 pt) Calculate $\Delta\hat{N}$, the uncertainty in photon number.

(g – 1.5 pt) Suppose that exactly one photon is removed from the field, possibly by leaking through one of the cavity mirrors and then being measured with a detector. The new state of the field can be determined by acting on $|\psi\rangle$ with the photon annihilation operator $\hat{a}$. Calculate $\hat{a}|\psi\rangle$, and then renormalize your answer. We will let this new state, when properly normalized, be called $|\phi\rangle$; you are to use this new state in the remaining questions.

(h – 1 pt) Calculate $\langle\hat{N}\rangle$ for the state $|\phi\rangle$.

(i – 0.5 pt) What is the probability of measuring that there are exactly 2 photons in the state $|\phi\rangle$?

Fall 2015 Comprehensive Exam
511R

The Hamiltonian for an electron at rest in a uniform and constant magnetic field oriented along the $\hat{z}$ axis (B_z) can be written as:

$$\hat{H} = -\gamma B_z \hat{S}_z \tag{1}$$

where γ is the gyromagnetic ratio. Suppose at time $t = 0$ the electron is in the spin state:

$$\chi = a_1 \begin{pmatrix} i \\ 2 \end{pmatrix}$$

This state vector is written in the basis of eigenstates of the $\hat{S}_z$ operator.

a. (1 pt) Solve for the normalization coefficient a_1.

b. (1 pt) Write the 2x2 matrix associated with the $\hat{S}_z$ operator.

c. (1 pt) What is the energy difference between the two energy eigenstates?

d. (1 pt) If a measurement of the angular momentum along the z direction is performed, what is the probability of measuring $+\hbar/2$?

e. (1 pt) Write an expression for the *time dependent* state vector, $\chi(t)$.

f. (2 pts) Calculate the expectation value, $\langle S_z \rangle$, of the electron spin angular momentum along the z axis for all times t.

g. (3 pts) Calculate the expectation value, $\langle S_x \rangle$, of the electron spin angular momentum along the x axis for all times t.

Recall that:

$$\hat{S}_x = \frac{\hbar}{2} \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \tag{2}$$

Fall 2015 Written Comprehensive Examination
OPTI-537

Answer the following questions related to solid-state physics and semiconductor detectors. All parts weighted as indicated.

(a) (1 pt) Explain what is meant by a Fermion. Write a properly normalized two-electron wave function with both electrons in a $\psi_{1s}(\mathbf{r})$ ground state if the up and down spin states are represented by α and β. Show that this two-electron wavefunction is consistent with the Pauli exclusion principle.

(b) (2 pts) If a macroscopic crystal with an orthorhomic lattice structure with primitive direct lattice constants a = 3 Å, b = 4 Å, c = 5 Å has dimensions of .9 mm x .8 mm x 1 mm in the $\hat{\mathbf{x}} = \hat{\mathbf{a}}$, $\hat{\mathbf{y}} = \hat{\mathbf{b}}$, and $\hat{\mathbf{z}} = \hat{\mathbf{c}}$ directions respectively, and we invoke Bloch's theorem and Born-Von Karman boundary conditions, how many electrons fit inside a single band in the 1st Brillouin zone of this crystal?

(c) (2 pts) Then consider a 2D square lattice with primitive lattice constants a = b = 5 Å. Use the Ewald sphere construction to work out what the allowed elastic reflections ($\mathbf{k_{out}}$) (expressing them in terms of $\hat{\mathbf{A}}$ and $\hat{\mathbf{B}}$) are if light has an incident **k** vector (in units of Å^{-1})

$$\mathbf{k}_{inc} = \frac{4\pi}{5}\hat{\mathbf{A}} + \frac{2\pi}{5}\hat{\mathbf{B}}$$

where $\hat{\mathbf{A}}$ and $\hat{\mathbf{B}}$ are the unit vectors in reciprocal space corresponding to directions $\hat{\mathbf{a}}$ and $\hat{\mathbf{b}}$ in real space.

(d) (1pt) If a one-electron state of energy ε_1 has a probability of occupancy of .25 at some temperature T, what is the probability of occupancy (at the same temperature) of another one-electron state of energy $\varepsilon_2 = 2\varepsilon_1 - \varepsilon_F$ where ε_F is the Fermi energy?

(e) (2 pts) Make a sketch that shows the concentrations of majority and minority charge carriers versus temperature ranging from 0K to 500K for typical crystals of doped semiconductor. Annotate the plot with an explanation of what physical effects are being revealed.

(f) (2 pts) Make a sketch of the basic structure (cross-section) of a CCD detector that explains how the device functions as a photodetector and readout. Add a diagram that illustrates back-thinning and explain what it is used for.

Fall 2015 Written Comprehensive Examination
OPTI 537

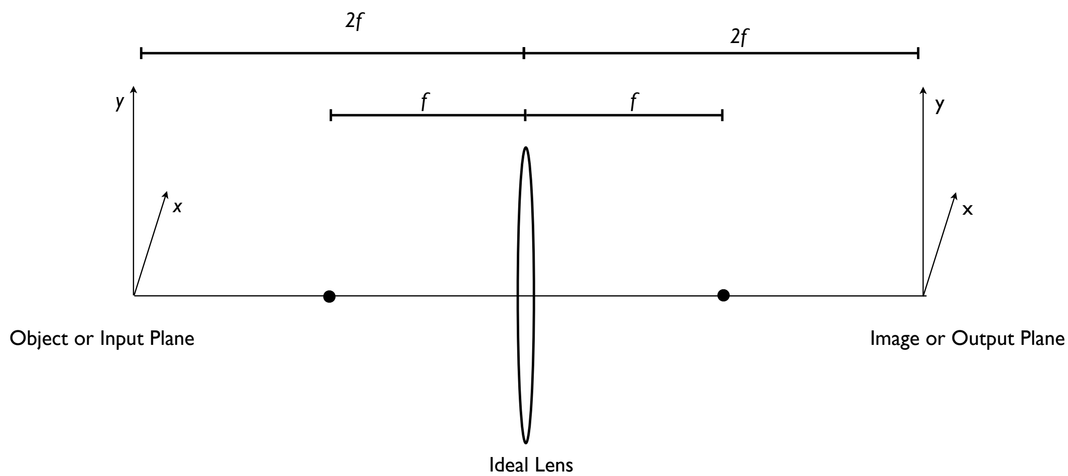
Consider imaging systems that are imaging coherent, monochromatic objects with wavelength λ. Fresnel diffraction for a system described by an ABCD matrix is given by:

$$u_{out}(\vec{r}) = \frac{-i}{B\lambda} \int_{-\infty}^{\infty} d^2 r_0 \, u_{in}(\vec{r_0}) \exp\left(\frac{i\pi}{\lambda B} [A|\vec{r_0}|^2 + D|\vec{r}|^2 - 2\,\vec{r}\cdot\vec{r_0}]\right) \qquad Eq\ (1)$$

where both $\vec{r}$ and $\vec{r_0}$ are 2 vectors in the x-y plane. (A selection of ABCD matrices is given in the table on the next page.)

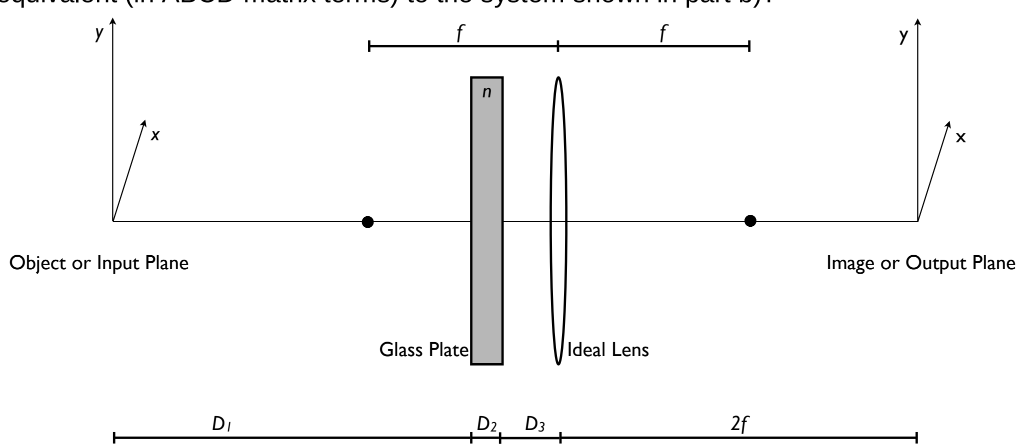
(a) (3 points) Show that Equation 1 above simplifies to standard Fresnel transform when considering free-space propagation.

(b) (3 points) Now consider the simple imaging system below,



Write down the ABCD matrix for this system and describe how to use Equation 1 to describe the propagation of a field from the input plane to the output plane.

(c) (4 points) Finally, consider the following system where we have inserted a piece of glass with index of refraction *n* before the lens. The distance from the lens to the image plane remains *2f*. However the distances D_1, D_2, and D_3 no longer add up to *2f*. Compute the ABCD matrix for this system. What condition must be true for this imaging system to be equivalent (in ABCD matrix terms) to the system shown in part b)?



ABCD Matrics:

Propagation in a medium of constant refractive index	$\begin{bmatrix} 1 & d \\ 0 & 1 \end{bmatrix}$	d = distance along optical axis
Refraction at a flat interface	$\begin{bmatrix} 1 & 0 \\ 0 & \frac{n_1}{n_2} \end{bmatrix}$	n1 = initial refractive index n2 = final refractive index
Refraction by a thin lens	$\begin{bmatrix} 1 & 0 \\ -\frac{1}{f} & 1 \end{bmatrix}$	F = focal length (f>0 means convex lens)

Fall 2015 Written Comprehensive Exam
OPTI 544

In this problem we consider the electromagnetic field of a single mode of an optical cavity. The field has an optical frequency ω. The state of the field is described in terms of photon number states $|n\rangle$, where n represents the number of photons in state $|n\rangle$. These states are eigenstates of the photon number operator $\hat{N}$ such that $\hat{N}|n\rangle = n|n\rangle$, and therefore they are also eigenstates of the field Hamiltonian $\hat{H} = \hbar\omega\hat{N} + \hbar\omega/2$. Below, you will work with $\hat{N}$ as well as the photon annihilation operator $\hat{a}$. Recall that $\hat{a}|n\rangle = \sqrt{n}\,|\,n-1\rangle$ for $n > 0$, and $\hat{a}|0\rangle = 0$.

For the following questions, you will use a superposition of photon number states that is defined as

$$|\psi\rangle = \frac{1}{2}|0\rangle + c|2\rangle + \frac{i}{2}|4\rangle,$$

where c is a scalar.

(a – 1.5 pt) Specify a number for c so that $|\psi\rangle$ is properly normalized. You will use this number for c in the remainder of this problem.

(b – 1 pt) Is $|\psi\rangle$ an energy eigenstate of the field? Briefly justify your answer.

(c – 1 pt) Suppose you could measure the number of photons in the field. Specify the probabilities of exactly measuring 0, 1, 2, 3, and 4 photons.

(d – 1 pt) Calculate $\langle\hat{N}\rangle$, the expectation value for the number of photons in the field.

(e – 1 pt) Calculate $\langle\hat{N}^2\rangle$.

(f – 1.5 pt) Calculate $\Delta\hat{N}$, the uncertainty in photon number.

(g – 1.5 pt) Suppose that exactly one photon is removed from the field, possibly by leaking through one of the cavity mirrors and then being measured with a detector. The new state of the field can be determined by acting on $|\psi\rangle$ with the photon annihilation operator $\hat{a}$. Calculate $\hat{a}|\psi\rangle$, and then renormalize your answer. We will let this new state, when properly normalized, be called $|\phi\rangle$; you are to use this new state in the remaining questions.

(h – 1 pt) Calculate $\langle\hat{N}\rangle$ for the state $|\phi\rangle$.

(i – 0.5 pt) What is the probability of measuring that there are exactly 2 photons in the state $|\phi\rangle$?

Fall 2015 Comprehensive Exam
544

The Hamiltonian for an electron at rest in a uniform and constant magnetic field oriented along the $\hat{z}$ axis (B_z) can be written as:

$$\hat{H} = -\gamma B_z \hat{S}_z \tag{1}$$

where γ is the gyromagnetic ratio. Suppose at time $t = 0$ the electron is in the spin state:

$$\chi = a_1 \begin{pmatrix} i \\ 2 \end{pmatrix}$$

This state vector is written in the basis of eigenstates of the $\hat{S}_z$ operator.

a. (1 pt) Solve for the normalization coefficient a_1.

b. (1 pt) Write the 2x2 matrix associated with the $\hat{S}_z$ operator.

c. (1 pt) What is the energy difference between the two energy eigenstates?

d. (1 pt) If a measurement of the angular momentum along the z direction is performed, what is the probability of measuring $+\hbar/2$?

e. (1 pt) Write an expression for the *time dependent* state vector, $\chi(t)$.

f. (2 pts) Calculate the expectation value, $\langle S_z \rangle$, of the electron spin angular momentum along the z axis for all times t.

g. (3 pts) Calculate the expectation value, $\langle S_x \rangle$, of the electron spin angular momentum along the x axis for all times t.

Recall that:

$$\hat{S}_x = \frac{\hbar}{2} \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \tag{2}$$

Fall 2015 Written Comprehensive Exam
Opti 546

There is no need for excessive formulae to answer the six parts of this question, you may simply state equations you feel are relevant.

(a - 1pt) Write down the ray transform that relates the incident ray vector $\binom{x}{x'}_i$ to the final ray vector $\binom{x}{x'}_f$ across a first-order optical system.

(b - 1pt) Prove that for an afocal or telescopic first-order optical system collimated rays at the input are converted into collimated rays at the output.

(c - 2pts) Using a sketch and accompanying words describe what is meant by the term Airy disk as applied to the problem of diffraction of a plane-wave field by a circular aperture.

(d - 2pts) Write down the equation of motion for the Lorentz electron oscillator model describing an atom in interaction with a light field and clearly identify the terms that incorporate the effects of any externally applied electric fields and Coulomb interactions.

(e - 2pts) Write down the general form of the (3×3) matrix representation of the force constant tensor K_{ij} in the principal axes system for the cases of isotropic, uniaxial, and biaxial crystals.

(f - 1pt) In electro-optics a static electric field E_{DC} applied to a medium can be used to alter the refractive indices experienced by two orthogonal linear field polarization states, resulting in a refractive-index difference $\Delta n(E_{DC})$. Describe what distinguishes the electro-optical Pockels and Kerr effects in terms of the variation of $\Delta n(E_{DC})$ with the applied static electric field.

(g - 1pt) Following on from part (f), is it possible to observe the Pockels effect in an isotropic medium?

Fall 2015 Written Comprehensive Exam
Opti 546

This problem explores ray and Gaussian beam propagation across an optical system. The ABCD law for the evolution of the complex beam parameter $\frac{1}{q} = \frac{1}{R} + \frac{2i}{kw^2}$ across the optical system is

$$\frac{1}{q_1} = \frac{C + D/q_0}{A + B/q_0}, \qquad M_L = \begin{pmatrix} 1 & L \\ 0 & 1 \end{pmatrix}, \qquad M_f = \begin{pmatrix} 1 & 0 \\ -1/f & 1 \end{pmatrix},$$

M_L and M_f being the ray transfer matrices for free-space and a thin lens.

(a - 3pts) A non-imaging optical system which is prescribed by a ray transfer matrix with elements A, B, C, D may be realized using a lens of focal length f_1 followed by a section of free-space of length B and finally a thin lens of focal length f_2. Find expressions for the focal lengths f_1 and f_2 in terms of the ray matrix elements A, B, C, D so that the above statement is true.

(b - 2pts) For the remainder of this question we consider a telescopic or afocal optical system with $A = 2, B = 20$ cm. Using your results from part (a) describe and sketch an optical system that would realize this optical system making sure to quote the required values for f_1 and f_2 in cm.

(c - 2pts) Consider that an image represented by collimated rays is incident on the optical system of part (b). Using arguments based on ray transformations describe the form of the image at the output.

(d - 2pts) Next consider that a collimated Gaussian beam of incident spot size w_0 is incident at the input to the optical system from part (b). Assuming that $|A| >> |B|/z_R$, with $z_R = kw_0^2/2$ the Rayleigh range of the incident Gaussian beam, derive expressions for the output inverse radius of curvature $(1/R_1)$ and the ratio (w_1/w_0) of the output and incident spot sizes.

(e - 1pts) Discuss the relation between your results from parts (c) and (d).