Spring 2014 Written Comprehensive Exam
Opti 501

System of units: MKSA

All fields and all parameters in this problem are real-valued; do not use the complex notation.

The E-field of a monochromatic plane-wave propagating in free-space is given by

$$\boldsymbol{E}(\boldsymbol{r},t) = E_{x0}\cos(k_0 z - \omega t + \varphi_{x0})\,\hat{\boldsymbol{x}} + E_{y0}\cos\big(k_0 z - \omega t + \varphi_{y0}\big)\,\hat{\boldsymbol{y}}. \qquad (1)$$

1 Pt a) Identify the k-vector (both direction and magnitude), and relate it to the parameters of Eq. (1).

2 Pts b) Describe the state of polarization of the beam in terms of the values of $E_{x0}, \varphi_{x0}, E_{y0}, \varphi_{y0}$. (For example, describe the circumstances under which the beam would be linearly-polarized, or circularly-polarized, etc.)

2 Pts c) Use one of Maxwell's equations to determine the H-field $\boldsymbol{H}(\boldsymbol{r},t)$ of the plane-wave in terms of the parameters of Eq. (1).

2 Pts d) Write the complete expression of the plane-wave's Poynting vector, $\boldsymbol{S}(\boldsymbol{r},t) = \boldsymbol{E}(\boldsymbol{r},t) \times \boldsymbol{H}(\boldsymbol{r},t)$, then explain its meaning and significance.

1 Pt e) Show that, for a circularly-polarized beam, the Poynting vector derived in part (d) is a constant, that is, it does *not* depend on z and t.

2 Pts f) Show that, for a linearly-polarized beam, the Poynting vector is a function of z and t. Thus, at a fixed instant of time, say, $t = t_0$, the Poynting vector $\boldsymbol{S}$ will have different values at different locations along the z-axis. Does this variation of $\boldsymbol{S}$ with z violate the law of conservation of energy? Explain.

A camera objective consists of two thin lenses in air. The first lens has a focal length of f_1=-30mm and a circular aperture diameter of 5mm. The first lens is the system aperture stop. The focal length of the second lens is f_2=30mm. The first lens is placed at the front focal point of the second lens. Light travels from left to right. The object is at infinity and subtends a field angle of ±10 degrees. A detector is placed at the image plane to capture the image.

(a) Determine the optical power and equivalent focal length of the camera objective. (1 point)

(b) Determine the back focal distance (BFD) of the camera objective and the minimum diameter of the image detector. (1 point)

(c) Determine the minimum lens diameters D2 for the lens 2 which avoids vignetting. (2 points)

(d) Locate the entrance pupil and exit pupil of the camera objective. (1 point)

(e) Determine the slope of the marginal ray in image space and determine the F/# of the combined objective lens. (1 point)

(f) Determine the Lagrange invariant of the camera objective. (1 point)

(g) Draw a neat diagram showing the optical axis and the locations of the two lenses, the stop, and the detector. On the same diagram, trace the chief ray and marginal ray through the system. (2 points)

(h) If the detector has pixel diameter of 10µm, determine the allowable detector displacement from the nominal image position before the resulting blur exceeds the detector's pixel size. (1 point)

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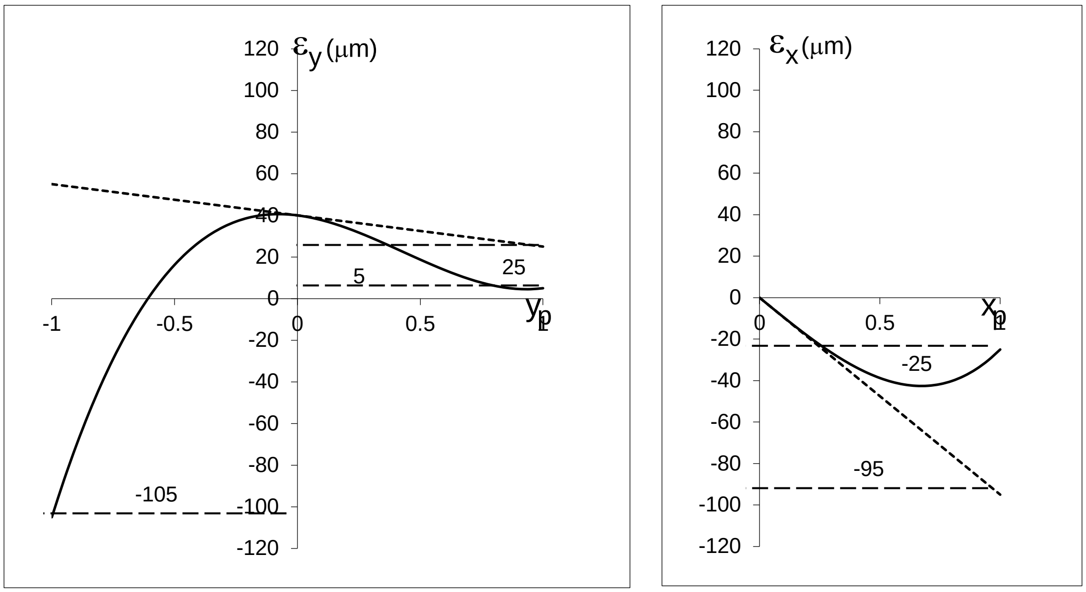
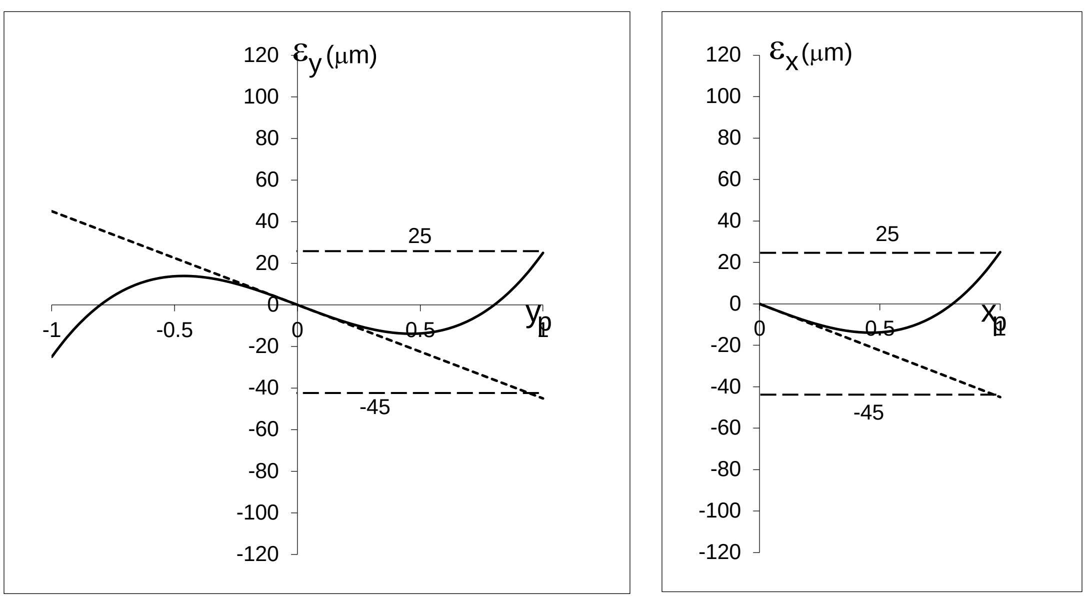
Opti 503

You are given the following set of ray fans. The ray fans are plotted at H = 0 and H = 1.0 for an f/5 system. The dashed lines indicate the slope of the rayfans through the origin. Only first and third order aberrations are present. Calculate the aberration coefficients W_{XYZ} for each aberration. The units on the plots are microns. Show your work.

H = 1



H = 0



Consider an interface at $z=0$ between a dielectric medium with index *n* and free space with radius r_0. An on-axis plane wave of wavelength λ is incident on this interface from the dielectric side. Assume that the interface is imperfect in that the interface actually occurs at $z=a\cos(2\pi\xi_0 x)$, and the region is limited by a circular aperture of diameter *d*. Assume that *a* is a small constant such that $(n-1)a \ll \lambda$, and assume that $d \gg \lambda$. This is a model for a rough surface.

- (6 pts) Compute diffracted irradiance in the Fraunhofer region interface. How does it differ from the perfectly smooth case (*a*= 0)?
- (4 pts) Now consider the case where the roughness has more than one spatial frequency and can be described as $z(x,y)=f_{rough}(x,y)$. How is the far-field diffraction pattern related to the roughness function? What can be said about the diffracted spot size if the roughness is limited to spatial frequencies below some maximum value ξ_1?

Recall that $e^{x_0} \approx 1 + x_0 + O\left(x_0^{\,2}\right)$.

Some potentially useful Fourier Transforms:

$$\cos(2\pi\xi_0 x) \leftrightarrow \frac{1}{2}\left[\delta(\xi-\xi_0)+\delta(\xi+\xi_0)\right]$$

$$\sin(2\pi\xi_0 x) \leftrightarrow \frac{1}{2j}\left[\delta(\xi-\xi_0)-\delta(\xi+\xi_0)\right]$$

$$\mathrm{cyl}\left(\frac{\sqrt{x^2+y^2}}{d}\right) \leftrightarrow \frac{\pi r_0^{\,2}}{4}\mathrm{somb}\left(r_0\sqrt{\xi^2+\eta^2}\right)$$

$$\mathrm{rect}\left(\frac{x}{a},\frac{y}{b}\right) \leftrightarrow ab\,\mathrm{sinc}(a\xi, b\eta)$$

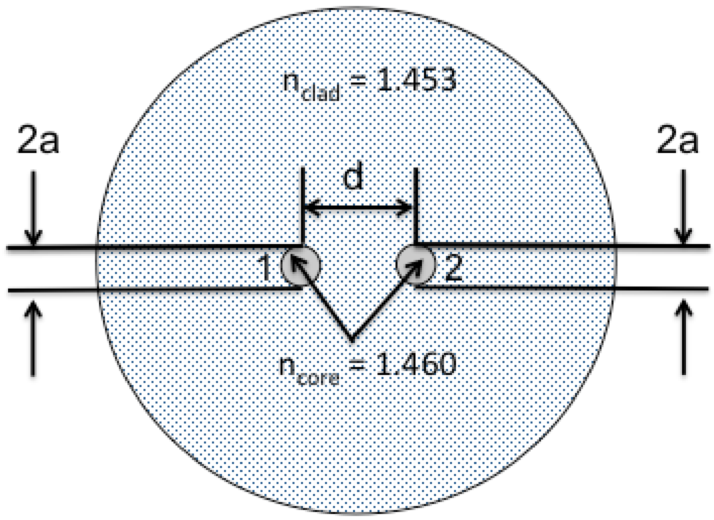
Spring 2014 Comprehensive Exam
OPTI 507

Consider an excitonic optical transition in GaAs. For simplicity, assume the conduction and valence band to be parabolic and isotropic with equal effective electron and hole masses, $m_e = m_h = 0.1m_0$ (where m_0 is the electron mass in vacuum). Assume the bandgap to be Eg=1.43eV and the exciton binding energy E_B=4.2meV. Sketch the 1s-exciton dispersion (i.e. the dispersion of the center of mass motion) and in the same plot the dispersion of light in the medium (refractive index n=3). Clearly label the axes and all special points. Indicate the point of possible strong absorption. Note that in this problem we neglect polariton effects, which may make it a little bit oversimplified. Write down the equation that would let you determine the numerical value of the wave vector *q* where strong absorption is possible. <u>Use the following further simplification to determine the *q*-value:</u> neglect the exciton's kinetic energy (this can be done by letting the exciton mass approach infinity). Determine the *q*-value and the corresponding wavelength in the medium.

(You may use $\hbar = 0.658$ meV ps .)

(10 points)

This problem concerns the optical fiber directional coupler shown below; light propagation occurs into the page. The fiber has two cores, each of which has a physical radius of 4μm, with a refractive index of n_{core} = 1.460 at a wavelength of 1.55μm. The surrounding cladding material has a refractive index of n_{clad} = 1.453 at 1.55μm. The cores are separated by a distance d that is large enough that you can ignore the presence of the other core when considering the modes of propagation pertaining to each core. You can assume that there is no propagation loss in either core.



a = 4 x 10^-4 cm (4μm)

a) What is the V parameter for each of the fiber cores at 1.55μm? (2 points)

b) Are the cores single mode at 1.55μm wavelength? Assuming that the refractive index has no dispersion, are the cores single mode at 1.31μm wavelength? (2 points)

c) Since the cores are identical, their propagation constants are identical and phase matched directional coupling can occur. Suppose we initially couple light of power P_0 into the core labeled "1" and no light into the core labeled "2". If the coupling length is given by L_0, sketch the power guided by each core vs. the propagation distance z for z between 0 and $4L_0$ (3 points).

d) We would like to make an all-optical switch using this directional coupler. Suppose the length, L_{bar} of the dual core fiber is such that at low powers all of the light entering core "1" exits at core "1" as well. If the nonlinear refractive index of the fiber (core and cladding) is given by n_2, determine an approximate expression for the minimum intensity, I_{cross} that must propagate in core "1" in order to get the light to now be output from core "2" (3 points). You may leave the result in terms of algebraic quantities (3 points).

Spring 2014 Written Comprehensive Exam
Opti 511R

A monochromatic and linearly polarized laser field of the form $E = \frac{1}{2}\hat{z}E_o e^{-i\omega t} + c.c.$ is turned on at time $t = 0$ and interacts with a gas cell of hydrogen atoms. The laser frequency is resonant with the $n = 1$ to $n = 3$ transition of hydrogen.

(a) [10%] To what quantum state $\psi_{n,l,m}$ will the atoms be excited?

(b) [30%] Given an electric field strength of $E_o = (\frac{3\pi\hbar}{ea_o}) \times 10^9 \approx 11.6 \times 10^4$ $[\frac{J}{C\cdot m}]$, calculate the time t_1 at which the laser field should be shut off to leave the maximum number of atoms in the state $n = 3$.

With the laser field now shut off, atoms will radiatively decay from this $n = 3$ state. Neglect non-radiative decay mechanisms such as collisions in the following problems.

(c) [20%] Make a sketch labeling the relevant energy levels involved (e.g. starting from the 1S ground state), and calculate all spontaneous emission wavelengths that would be observed (to 3 significant digits).

(d) [30%] Calculate the natural lifetime of this state.

(e) [10%] Of the atoms that were initially excited to the $n = 3$ state, what fraction will radiatively decay back to the ground state?

The following expressions may be useful for these calculations:

$$A = \left|\frac{\Delta E}{E_1}\right|^3 \times \frac{|\vec{\wp}|^2}{e^2 a_0^2} \times 2.35 \times 10^9 \text{ s}^{-1},$$

where A is the Einstein A coefficient, ΔE is the energy difference between two levels, and E_1 is the hydrogen ground-state energy of -13.6 eV.

$a_o = 0.5 \times 10^{-9} [m]$
$\langle l = 0, m = 0 \,|\hat{\mathbf{r}}|\, l = 1, m = 0\rangle = (0, 0, \sqrt{\frac{1}{3}})$
$\langle l = 0, m = 0 \,|\hat{\mathbf{r}}|\, l = 1, m = 1\rangle = (-\sqrt{\frac{1}{6}}, -i\sqrt{\frac{1}{6}}, 0)$
$\langle l = 0, m = 0 \,|\hat{\mathbf{r}}|\, l = 1, m = -1\rangle = (\sqrt{\frac{1}{6}}, -i\sqrt{\frac{1}{6}}, 0)$

$\langle 1S|r|2P\rangle = 1.29a_0$
$\langle 1S|r|3P\rangle = 0.517a_0$
$\langle 2S|r|3P\rangle = 3.07a_0$
$\langle 2P|r|3S\rangle = 0.95a_0$

Spring 2014 Written Comprehensive Examination
OPTI 537

Answer the following questions related to radiative transport. The radiative transport equation (RTE), or Boltzmann equation, describes the time evolution of a phase-space distribution function w in terms of four processes: absorption, emission, propagation, and scatter

$$\frac{dw}{dt} = \left[\frac{\partial w}{\partial t}\right]_{abs} + \left[\frac{\partial w}{\partial t}\right]_{emiss} + \left[\frac{\partial w}{\partial t}\right]_{prop} + \left[\frac{\partial w}{\partial t}\right]_{scat} \qquad (1)$$

(Part A: 4 Points for answering the following bold questions)

What is w a function of and what are the units? What does *w* describe?

In class, we derived forms for each of the terms in the RTE and wrote an overall spatio-temporal-integro-differential transport equation of the form:

$$\frac{dw}{dt} = -c_m \mu_{total} w + \Xi_{p,E} - c_m \hat{\boldsymbol{s}} \cdot \nabla w + \boldsymbol{K} w \qquad (2)$$

where ***K*** is an integral operator. Please associate each term in equation (1) with the terms in equation (2). **Describe and write the units of each of the variables in equation (2).**

What is meant by the "steady-state solution" of the RTE? What changes in equation (2) when we assume steady state?

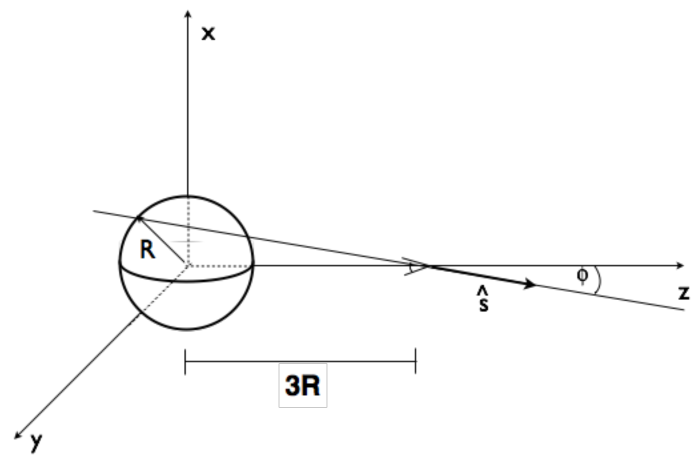
Write the steady-state equation for a non-absorbing and non-scattering medium.

(Part B: 6 Points)

Now assume that we have an incoherent, uniform spherical source of radius *R* centered at the origin that isotropically emitting photons of energy *E* at a rate of *A* photons per second. These photons are propagating in a non-absorbing and non-scattering medium. We are observing photons at a distance of *3R* away from the origin along the *z* axis (see Figure below). **What is the steady-state phase-space distribution function at this location for photons traveling in the *z* direction (phi = 0)?**

Building on the above question. **What is the steady-state phase-space distribution function at the observation point for photons traveling at an angle of $\phi=15^o$?** You may leave your answer in terms of ϕ if you would like.

What is the steady-state phase-space distribution function at the observation point for photons traveling at an angle of $\phi=45^o$?


x
R
$\hat{s}$
φ
z
3R
y

Spring 2014 Written Comprehensive Exam
OPTI 544

In this problem we explore a quantum version of the electron oscillator model, i. e. an atom where the internal motion of the electron relative to the nucleus is described by a one-dimensional quantum mechanical harmonic oscillator, instead of the classical harmonic oscillator used in the Lorentz model. The quantum oscillator has frequency ω_0, creation and annihilation operators $b^\dagger, b$, and position operator $\hat{x} = x_0\left(b^\dagger + b\right)$.

(a) Sketch the shape of the wave functions (probability amplitude $\varphi_n(x)$ as function of electron position x) for the ground state $|n=0\rangle$ and first excited state $|n=1\rangle$. Next, sketch the probability density $P(x)$ for the superposition $\psi(x) = \frac{1}{\sqrt{2}}\left(\varphi_0(x) + e^{-i\omega_0 t}\varphi_1(x)\right)$, at times when $\omega_0 t = 0,\ \pi/2,\ \pi$. What do these probability densities tell us about the dipole moment as function of time? Note: Only the qualitative shapes of the wave functions and probability densities are needed; there is no need to recall their exact functional form or to plot them in a way that is quantitatively accurate. (30%)

(b) Now consider the dipole matrix elements $p_{mn} = \langle m|\hat{p}|n\rangle$ between the harmonic oscillator states. What are the selection rules for electric-dipole transitions, i. e. which p_{mn}'s are zero and which are non-zero? Write out the corresponding matrix for the dipole operator in the basis of oscillator states $|n\rangle$, for $n \leq 5$. (30%)

We put the quantum electron oscillator inside an optical cavity, where it interacts with a quantized mode of the electromagnetic field with frequency ω, creation and annihilation operators $a^\dagger, a$, and electric field operator $\hat{E} = E_0\left(a^\dagger + a\right)$. For simplicity, we assume the electric field is parallel to the electron oscillator motion.

(c) Write down the Hamiltonian for the system consisting of the electron oscillator and the quantized field mode, including the electric dipole interaction. Compare to the Jaynes-Cummings Hamiltonian for a two-level atom coupled to a quantized mode of the electromagnetic field. (20%)

(d) Now assume $\omega = \omega_0$. Based on what you found in (c), what do you expect will happen if the system at $t = 0$ has one quantum of excitation in the electron oscillator, and zero quanta of excitation in the field? Sketch the populations of the states $|1_{osc}\rangle|0_{field}\rangle$ and $|0_{osc}\rangle|1_{field}\rangle$ as a function of time. (20%)

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OPTI 546 Spring 2014

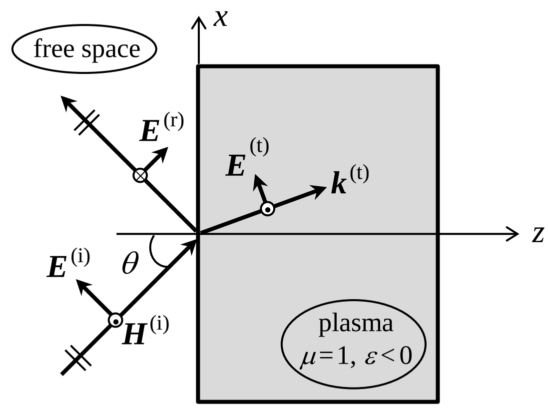
There is no need for excessive formulae to answer these questions, you may simply state equations you feel are relevant.

1. **What distinguishes the geometric ray path that joins an initial point at the input of an optical system to a final point at the output of the optical system, as opposed to any other path that joins the initial and final points? (1pt)**
2. **Provide two alternate names for the function that yields the field at the output of an optical system due to a point source at the input. (1pt)**
3. **Write down the Lorentz harmonic oscillator model for the displacement of the electron cloud center with respect to the nucleus in an atom subject to an external optical field, and identify the terms in the equation that incorporate the effects of Coulomb interactions and transverse fields. (3pts)**
4. **Give a brief description of the additional physics that must be added to the Lorentz harmonic oscillator model above to allow for the treatment of second harmonic generation. (2pts)**
5. **The second-order nonlinear polarization is given by $P^{(2)}(t) = \epsilon_0 \chi^{(2)} E^2(t)$. For the case of a monochromatic fundamental field of frequency ω demonstrate that this nonlinear polarization provides a source of radiation at both zero frequency (optical rectification) and at the second-harmonic at 2ω. (3pts)**

Spring 2014 Written Comprehensive Exam
Opti 501

System of units: MKSA

A *p*-polarized monochromatic plane-wave arrives from free-space at the flat surface of a plasma at an oblique angle θ, as shown. The optical properties of the plasma are specified by its permittivity $\varepsilon(\omega)$, a real-valued *negative* entity, and by its permeability $\mu(\omega) = 1$.



2 Pts a) Write expressions for the E and H fields of the incident beam as functions of space and time.

2 Pts b) Write expressions for the E and H fields of the reflected beam as functions of space and time.

2 Pts c) Write expressions for the E and H fields of the beam transmitted into the plasma as functions of space and time. Identify the real and imaginary components of the k-vector, and relate them to the various parameters of the system.

2 Pts d) Match the boundary conditions at the plasma surface, and obtain expressions for the Fresnel reflection and transmission coefficients ρ_p and τ_p, respectively.

2 Pts e) Show that the reflectivity of the plasma is always 100%, irrespective of the incidence angle θ, or of the exact value of $\varepsilon(\omega)$. Explain the apparent contradiction between a 100% reflectance at the surface and the existence of electromagnetic field energy inside the plasma.

In 2013, Nokia announced the Lumia 1020 Windows 8 smartphone featuring a next generation 41MP sensor. The new sensor has total sensor pixels of 7712x5360=41.3MP and the pixel size is 1.1 micron. The F-number of the lens is F/2.2. Assume the lens is an ideal thin lens without thickness and the distance from the lens to the sensor is 4mm when the object is 5 meters away from the smart phone.

1. What is the focal length of the lens? (1 point)
2. What is the aperture size? (1 point)
3. Draw a diagram with object, stop, lens, and image; indicate the chief and marginal rays. (1 point)
4. Assume a landscape lens (a camera lens with the stop as the first element) is used in this smart phone. Where are you going to move the stop to achieve telecentricity in sensor side? (1 point)
5. To achieve the resolution of the sensor, what should the F-number of the lens be? (1 points)
6. What are the total effective pixel numbers with the current F/2.2 lens? (2 points)
7. If the object is moved 1 meter towards the smart phone which is stationary, how should the lens be moved to obtain a good image of the object? What is the distance between the sensor and the lens? (3 points).

Spring 2014 Comprehensive Exam
OPTI 503

The local planetarium is showing a laser light show. They have three lasers to create the special effects. The wavelengths of these lasers are 480 nm, 540 nm and 630 nm and each laser has the same maximum power. You can assume the screen reflects uniformly across the visible spectrum. A table of the CIE 2° Color Matching Functions is provided on the following page.

(a) What are the chromaticity coordinates of each laser?

(b) The three laser spots are overlapped to create a new color. If the blue laser provides 1 Watt, what powers do the red and green lasers need to put out to create a white spot? Assume the equal energy white (x_w = 0.333, y_w = 0.333).

λ (nm)	$\bar{x}(\lambda)$	$\bar{y}(\lambda)$	$\bar{z}(\lambda)$
380	0.00137	0.00004	0.00645
390	0.00424	0.00012	0.02005
400	0.01431	0.00040	0.06785
410	0.04351	0.00121	0.20740
420	0.13438	0.00400	0.64560
430	0.28390	0.01160	1.38560
440	0.34828	0.02300	1.74706
450	0.33620	0.03800	1.77211
460	0.29080	0.06000	1.66920
470	0.19536	0.09098	1.28764
480	0.09564	0.13902	0.81295
490	0.03201	0.20802	0.46518
500	0.00490	0.32300	0.27200
510	0.00930	0.50300	0.15820
520	0.06327	0.71000	0.07825
530	0.16550	0.86200	0.04216
540	0.29040	0.95400	0.02030
550	0.43345	0.99495	0.00875
560	0.59450	0.99500	0.00390
570	0.76210	0.95200	0.00210
580	0.91630	0.87000	0.00165
590	1.02630	0.75700	0.00110
600	1.06220	0.63100	0.00080
610	1.00260	0.50300	0.00034
620	0.85445	0.38100	0.00019
630	0.64240	0.26500	0.00005
640	0.44790	0.17500	0.00002
650	0.28350	0.10700	0.00000
660	0.16490	0.06100	0.00000
670	0.08740	0.03200	0.00000
680	0.04677	0.01700	0.00000
690	0.02270	0.00821	0.00000
700	0.01136	0.00410	0.00000
710	0.00579	0.00209	0.00000
720	0.00290	0.00105	0.00000

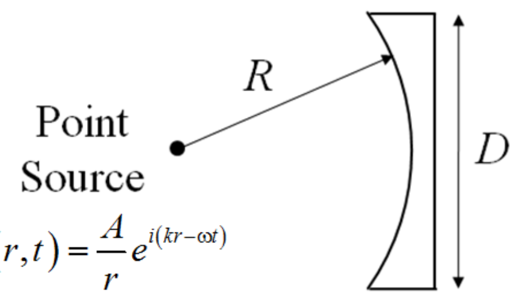
Opti505R Spring 2014

a) (2.5 pts) An ideal point source in air with λ = 0.5μm is placed at the center of curvature of a spherical mirror, as shown below, where R = 50 mm and D = 25 mm. Call the line between this position of the source and the mirror vertex the centerline. Describe the interference pattern generated between the point-source wave and the wave reflected by the mirror in a region between the source and the mirror. Include in your analysis fringe spacing and shape, if fringes are present. You may assume that the mirror is a perfect electrical conductor, and that scattering from edges of the mirror can be neglected.

b.) (2.5 pts) Repeat your analysis for (a) for the region to the left-hand side of the source.

c.) (2.5 pts) The source is moved vertically a distance of 100 μm upward. Where is the location of the image of the source?

d.) (2.5 pts) Describe the interference pattern generated between the point-source wave and the wave reflected by the mirror fro condition (c) in a region between the source and the mirror and on the left-hand side of the source. Include in your analysis fringe shape, if fringes are present, and a discussion of fringe intecepts with the centerline .



$$U(r,t)=\frac{A}{r}e^{i(kr-\omega t)}$$

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Spring 2014 Comprehensive Exam
OPTI 507

Consider a semiconductor quantum well. Assume you want to design a structure where the lowest intersubband transition (i.e. the transition between the lowest two subbands) corresponds to a frequency of 1 THz ($=1\times10^{12}$ Hz). Determine the dependence of the quantum well thickness L_z on the effective electron mass m_e, i.e. the function $L_z(m_e)$. Sketch this function. For the effective mass value of $m_e = 0.06m_0$ (where m_0 is the electron mass in vacuum) determine the value of L_z.

Instructions. If you remember the formula for the confinement energies, you can use it without proving it. If you don't remember that formula, you can get the confinement energies easily from the fact that the envelope functions of the lowest two subbands are $\xi_1(z) \sim \cos(\pi z / L_z)$ and $\xi_2(z) \sim \sin(2\pi z / L_z)$.

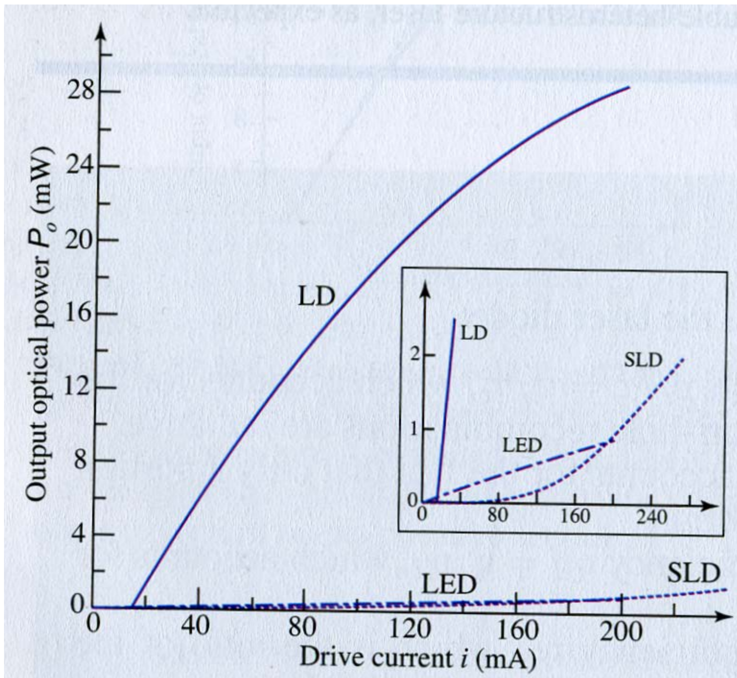
(You may use $\hbar = 0.658$ meV ps and $\hbar^2 / (2m_0) = 3.81\times10^{-16}$ eV cm^2.)

(10 points)

Spring 2014 Written Comprehensive Exam
OPTI 510

Problem 1: Photon sources, spontaneous and stimulated emission, and fiber optics

The light-current curves for a light-emitting diode (LED), superluminescent diode (SLD) and laser diode (LD) are shown. All three devices are InGaAsP/InP MQW structures operated at a wavelength of 1600 nm. The inset shows an expanded view of the curves.



(a) Sketch the emission spectrum (Intensity vs. Wavelength) for the LED, SLD and LD at high drive current (i ~ 200 mA). Explain the mechanism that leads to the different shape of the spectrum based on the light-current curves. (3 points)

(b) Sketch the emission spectrum for the LED, SLD and LD at low drive current (i ~ 1mA). Explain the shape and symmetry or lack of symmetry. (3 points)

(c) Consider the laser diode operating at 200mA. How many electrons are injected into the junction to extract a single photon? (2 points)

(d) Consider a fiber optic link, operating at the attenuation limited case and with negligible dispersion. The receiver sensitivity is -10dBm, the input and output couplers each has a power loss of 3dB and the attenuation coefficient of the fiber is 0.5dB/km. Determine the maximum length of the link assuming you are using the laser diode operating at 200mA. (2 points)

Spring 2014 Written Comprehensive Exam
Opti 511R

An optical cavity for a HeNe laser (633nm) is being constructed using a flat output coupler ($R_1 = \infty$) and a spherical mirror (R_2=50 cm). The output coupler has a power reflection coefficient of 99% while the spherical mirror is a 100% reflector. The cavity length is initially set at 50 cm. A micrometer adjustment with a range of $\pm 500\mu$m enables fine adjustment (ΔL) of the total cavity length (given by $L = 50\text{cm} + \Delta L$).

(a) [40%] If an output beam radius of only $w = 50\mu$m is desired, calculate the required micrometer adjustment ΔL to 2 significant digits (be sure to indicate the appropriate sign for $|\Delta L|$).

(b) [10%] What will be the beam radius on the spherical mirror?

(c) [30%] If the length of the HeNe gain tube is 10 cm, calculate the small-signal gain coefficient required for lasing.

(d) [10%] With the laser operating well above threshold, do you expect single frequency or multiple frequency operation? Explain why in a few sentences.

(e) [10%] If the laser is mode-locked, what will be the period between pulses?

Spring 2014 Written Comprehensive Examination
OPTI-537

Answer the following questions related to solid-state physics and semiconductor detectors. All parts weighted as indicated.

(a) (10%) Name the three Bravais lattices that have cubic symmetry and draw sketches of the unit cell for each, indicating all equivalent positions.

(b) (10%) Explain what is meant by the 1st Brillouin zone, and write an expression for its volume for one of the lattices of question (a) above if the direct primitive lattice vectors have a length of 5 Å.

(c) (10%) If a macroscopic crystal of one of the lattice structures in (a) above has dimensions of 1mm x 2mm x 5mm in the x, y, and z directions respectively, and we invoke Bloch's theorem and Born-Von Karman boundary conditions, what is the spacing between the allowed k_x, k_y, and k_z states? (Express your answer in the customary units of cm^{-1})

(d) (10%) What is the significance of the reciprocal lattice when, for example, one is writing a Fourier representation for a function that obeys the periodicity of the lattice. An example is the potential energy, which obeys $V(\mathbf{r}) = V(\mathbf{r}+\mathbf{R})$. Write such a Fourier representation for $V(\mathbf{r})$.

(e) (10%) In a few brief sentences, explain the fundamental assumptions and differences of the free-electron, nearly free electron, and tight-binding models of solid-state electronic structure.

(f) (10%) What is the formula for the Fermi-Dirac distribution and what does it describe? Using a diagram, illustrate its importance in determining whether a material is an insulator or a semiconductor?

(g) (10%) Using a set of plots and the equations of electrostatics, summarize how the depletion region and internal potential barrier in a PN junction form as N type and P type materials are brought together.

(h) (15%) Draw the band structure of an unbiased PN junction. Sketch the locations of donor and acceptor dopant states relative to the conduction and valence bands, and indicate where the Fermi level is at room temperature. Label which side is P and which is N and clearly indicate majority and minority carriers. Finally, clearly diagram and label the 2 electron and 2 hole currents present that exactly balance when the diode is in thermal equilibrium.

(i) (15%) Make a sketch of the diode current versus voltage (IV) curve, indicate which of the currents from (g) dominate in the reverse and forward biased regions. Explain why photodetectors in CMOS imaging sensors are normally operated in conditions of reverse bias.

Spring 2014 Written Comprehensive Exam
Opti 544

Consider a 50/50 quantum beamsplitter, with input ports 1 and 2, and output ports 3 and 4. The input state $|\psi_{in}\rangle$ can be expressed as a normalized superposition of states $|n\rangle_1|m\rangle_2$ where n labels the photon number in port 1, and m labels the photon number in port 2. The output state $|\psi_{out}\rangle$ can be similarly expressed.

(a – 2 pts) Which of the following (un-normalized) output states are product states, and which are entangled states?

(i) $(|0\rangle_3 + |1\rangle_3)(|0\rangle_4 + |1\rangle_4)$
(ii) $|0\rangle_3|1\rangle_4 + |1\rangle_3|0\rangle_4$
(iii) $|0\rangle_3|1\rangle_4 - |1\rangle_3|1\rangle_4$
(iv) $|0\rangle_3|0\rangle_4 + |1\rangle_3|1\rangle_4$

(b – 2 pts) Suppose that $|\psi_{in}\rangle = |2\rangle_1|0\rangle_2$. For this input state, the output state is

$$|\psi_{out}\rangle = \frac{1}{2}|2\rangle_3|0\rangle_4 + \frac{i}{\sqrt{2}}|1\rangle_3|1\rangle_4 - \frac{1}{2}|0\rangle_3|2\rangle_4.$$

If the number of photons in port 3 is measured but the number of photons in port 4 is not measured, what are the probabilities of obtaining 0, 1, and 2 photons in port 3? For each result of the measured photon number in port 3, give the corresponding state of port 4.

(c – 2 pt) Consider the output state given in (b). Suppose that the number of photons emerging from port 3 is measured by a device, but the result is immediately discarded so that no information about the port 3 measurement results is available. For this case, use the probabilities you found in part (b) to construct a formal mathematical description for the state of port 4.

(d – 4 pts) For $|\psi_{in}\rangle = |3\rangle_1|0\rangle_2$, evaluate $|\psi_{out}\rangle$. Remember that for a 50/50 beamsplitter, the relationships between the creation operators for the input and output ports can be written as $a_1^\dagger = \frac{1}{\sqrt{2}}(a_3^\dagger + ia_4^\dagger)$ and $a_2^\dagger = \frac{1}{\sqrt{2}}(ia_3^\dagger + a_4^\dagger)$.

OPTI 546 Spring 2014

This question deals with the Gaussian mode of optical resonators from two perspectives. First consider the perspective based upon the spot size $w(z)$ and radius of curvature $R(z)$ for a Gaussian beam propagating in free space

$$w(z) = w_0\sqrt{1+\frac{z^2}{z_0^2}}, \qquad R(z) = z + \frac{z_0^2}{z}, \qquad z_0 = \frac{\pi w_0^2}{\lambda}.$$

(a - 2pts) For a given focused Gaussian spot size w_0 plot the variation of the spot size $w(z)$ and radius of curvature $R(z)$ versus z over four Rayleigh ranges around the focus, making sure to indicate key features in your plots.

(b - 2pts) You are next given two mirrors, mirror 1 with radius of curvature $R_1 > 0$ that is placed at $z_1 \leq 0$, and mirror 2 with radius of curvature $R_2 > 0$ that is placed at $z_2 \geq 0$. What constraints, if any, are there on the values of R_1 and R_2 that can be used to obtain confined Gaussian solutions according to your results in part (a)?

(c - 1pts) Explain how you would use the results from part (a) to graphically solve for the possible mirror positions z_1 and z_2 and separations $L = (z_2 - z_1)$ that correspond to self-consistent Gaussian mode solutions of the optical resonator with a focused spot size w_0 at $z = 0$.

(d - 1pts) There is a special symmetric optical resonator that has only one possibility for the mirror separation L. Based on the above information give an explanation for what this special optical resonator is.

(e - 2pts) For the second perspective we consider the ABCD law for Gaussian beams $Q_{n+1} = \frac{C+DQ_n}{A+BQ_n}$ which maps the complex beam parameter $Q = \frac{1}{R} + \frac{2i}{kw^2}$ around successive round trips of an optical resonator described by a round-trip ray transfer matrix with real elements A, B, C, D. Show that the steady-state state complex beam parameter may be written in the form

$$Q_s = \frac{(D-A)}{2B} \pm \frac{\sqrt{(A+D)^2/4 - 1}}{B}.$$

(f - 2pts) Based on the solution from part (e) argue what form the complex beam parameter must take for a stable optical resonator that supports a confined Gaussian beam solution, and deduce the optical resonator stability condition, the steady state spot size w_s, and steady-state radius of curvature R_s in terms of the ray transfer matrix elements for the optical resonator.