

Summer 2017 Written Comprehensive Exam
Opti 501, day 1

System of units: MKSA (also known as SI)

1) A plane wave having wave-vector \mathbf{k} and frequency ω propagates in free space. Let the electric and magnetic fields of the plane-wave be written as

$$\mathbf{E}(\mathbf{r}, t) = \mathbf{E}_0 \exp[i(\mathbf{k} \cdot \mathbf{r} - \omega t)],$$

$$\mathbf{H}(\mathbf{r}, t) = \mathbf{H}_0 \exp[i(\mathbf{k} \cdot \mathbf{r} - \omega t)].$$

No sources are assumed to reside in free space; therefore, $\rho_{\text{free}}(\mathbf{r}, t) = 0$, $\mathbf{J}_{\text{free}}(\mathbf{r}, t) = 0$, $\mathbf{P}(\mathbf{r}, t) = 0$, and $\mathbf{M}(\mathbf{r}, t) = 0$.

- 2 Pts a) What does Maxwell's *first* equation have to say about the relation between \mathbf{E}_0 and \mathbf{k} ?
- 2 Pts b) What does Maxwell's *fourth* equation have to say about the relation between \mathbf{H}_0 and \mathbf{k} ?
- 2 Pts c) Apply Maxwell's *second* equation to derive a relation connecting \mathbf{E}_0 , \mathbf{H}_0 , \mathbf{k} , and ω .
- 2 Pts d) Apply Maxwell's *third* equation to derive another relation connecting \mathbf{E}_0 , \mathbf{H}_0 , \mathbf{k} , and ω .
- 2 Pts e) Combine the results obtained in parts (a)-(d) to eliminate \mathbf{E}_0 and \mathbf{H}_0 from the equations, thus arriving at the connection between \mathbf{k} and ω for plane-waves that travel in free space.

Hint: In part (e), the following vector identity will be helpful: $\mathbf{A} \times (\mathbf{B} \times \mathbf{C}) = (\mathbf{A} \cdot \mathbf{C})\mathbf{B} - (\mathbf{A} \cdot \mathbf{B})\mathbf{C}$.

Summer 2017 Written Comprehensive Exam
Opti 501, day 2

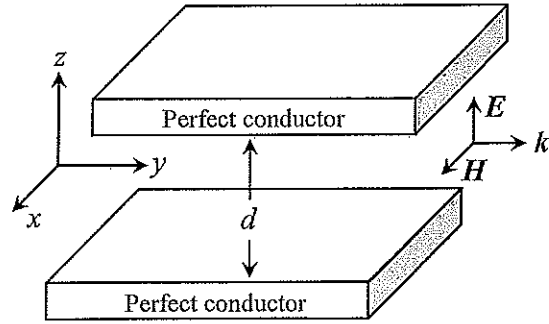
System of units: MKSA (also known as SI)

2) In the free-space region between two infinitely large, perfectly conducting parallel plates, an electromagnetic plane-wave propagates along the y -axis, as shown. The electric and magnetic fields of the plane-wave are given by

$$\mathbf{E}(\mathbf{r}, t) = E_0 \cos(k_0 y - \omega_0 t) \hat{z},$$

$$\mathbf{H}(\mathbf{r}, t) = H_0 \cos(k_0 y - \omega_0 t) \hat{x},$$

where E_0 and H_0 are the field amplitudes, k_0 is the propagation constant (also known as the wave-number), and ω_0 is the angular frequency of the plane-wave. All the above parameters, i.e., E_0 , H_0 , k_0 and ω_0 , are real-valued constants.



- 4 Pts a) Write Maxwell's equations in the region between the plates (i.e., $-\frac{1}{2}d < z < \frac{1}{2}d$), then find the relationship between k_0 and ω_0 on the one hand, and that between E_0 and H_0 on the other.
Hint: These relations should involve the speed of light in vacuum, $c = 1/\sqrt{\mu_0 \epsilon_0}$, and the impedance of free space, $Z_0 = \sqrt{\mu_0/\epsilon_0}$.
- 4 Pts b) Considering that the E and H fields inside the (perfectly conducting) plates must be zero, use Maxwell's boundary conditions to find the surface charge-density $\sigma_s(x, y, z = \pm \frac{1}{2}d, t)$ and the surface current-density $\mathbf{J}_s(x, y, z = \pm \frac{1}{2}d, t)$ on the inner surfaces of both plates.
- 2 Pts c) Show that the surface charge and current densities obtained in part (b) satisfy the charge-current continuity equation.
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Fall 2017 Qualifying and Comprehensive Exam

OPTI 502, Day 1

The lens in a cell phone camera can be modeled as a thin lens in air with the aperture stop at the thin lens. The lens is free of aberrations, with the following specifications:

$$\text{Focal length} = 5 \text{ mm.}$$

$$F/\# = f/2.8$$

The sensor in the camera has a diameter is 6 mm. The sensor is perpendicular to the optical axis of the thin lens and centered on the optical axis of the thin lens.

Determine:

- 1) The angular full field of view that the thin lens covers.
- 2) The entrance pupil diameter.
- 3) The location of the principal planes.
- 4) The geometrical depth of focus to keep the image blur no larger than 0.005 mm in diameter.

The owner of this cell phone is at a street fair and comes upon a chalk drawing on the sidewalk. A picture needs to be taken! The goal is to determine if the whole picture of the drawing will be in focus.

The person is standing and holding the cell phone at a height of 1.41 meters above the sidewalk. The cell phone is tilted down from the vertical 45 degrees to take a photo of the drawing. The drawing fills the field of view of the camera. The optical axis of the lens intersects the sidewalk at an angle of 45 degrees.

- 5) Make a drawing of the situation. The lens is imaging the drawing.
- 6) The image plane conjugate to the drawing is not perpendicular to the lens optical axis. Use first-order optics to determine the tilt angle of the image of the drawing with respect to the sensor plane.
- 7) Will this image plane tilt defocus the image at the sensor edge beyond the geometrical depth of focus of the lens? Assume that the camera is focused so that the image at the center of the sensor center is in proper focus.

Reasonable approximations may be made in the solution of this problem. Be sure to note any approximations made in the solution.

Fall 2017 Qualifying and Comprehensive Exam

OPTI 502, Day 2

An objective has been assembled using three thin lenses in air:

Lens 1: $f_1 = 20$ mm

Lens 2: $f_2 = -20$ mm

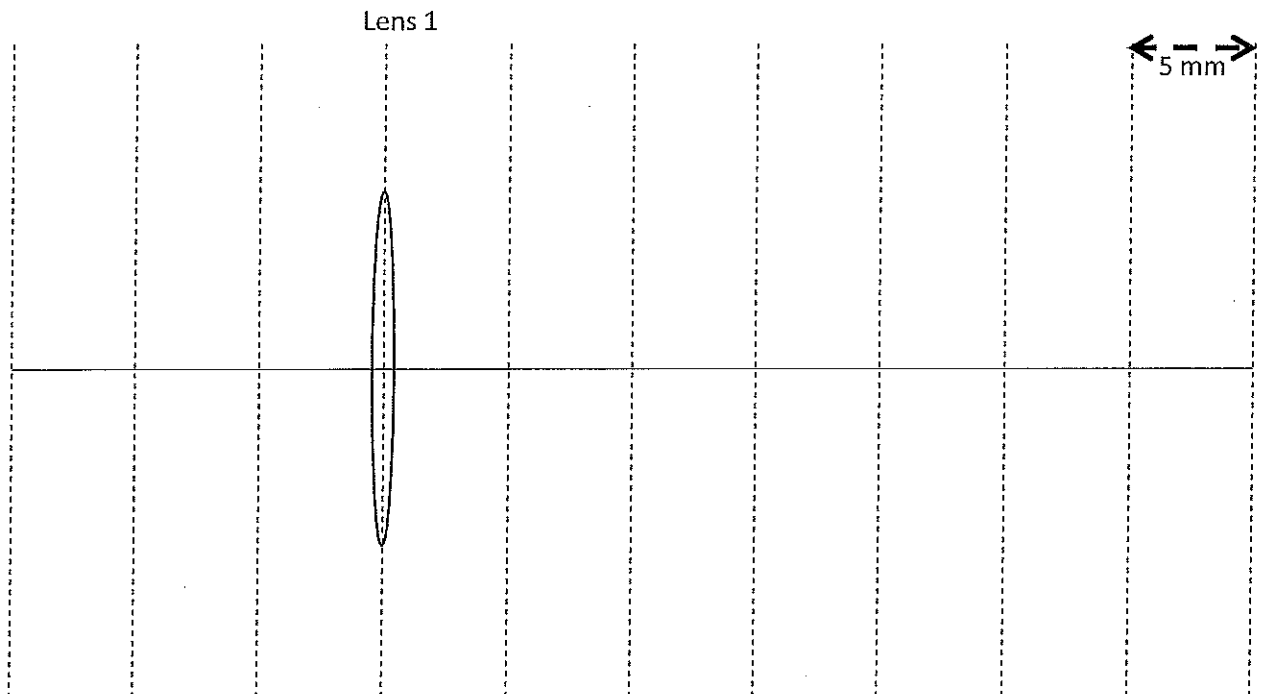
Lens 3: $f_3 = 20$ mm

Distance between lens 1 and lens 2: $t_1 = 10$ mm

Distance between lens 2 and lens 3: $t_2 = 5$ mm

Note: You must use the Gaussian methods in this problem set. No raytrace analysis is allowed.

a) Draw the triplet configuration and clearly mark "all" the distances (e.g., t_1 , BFD, t_{12} , etc...) used in your answers for problem (b) – (e). The lens 1 is already drawn in the left-side (towards object space). Draw the lens 2 and 3 to the right-side (towards image space). Light travels from left to right.



b) What is the system focal length, f ?

c) What is the system back focal distance, BFD? Draw the BFD and the rear focal point of the system on the drawing (a).

d) The system stop with 5 mm aperture diameter ($D_{\text{stop}} = 5 \text{ mm}$) is located halfway between the lens 1 and lens 2. Draw the Stop in the drawing (a).

e) Determine the location of entrance pupil (EP) and its diameter size D_{EP} . Draw the EP in the drawing (a) in correct scale with respect to D_{stop} size.

In this problem, a laser diode is tested that exhibits $\bar{\lambda} = 1550\text{nm}$ and has a Gaussian power spectrum width of approximately 12GHz. State any assumptions that you make.

(a) (4pts) The laser diode is coupled to an optical fiber that produces a Gaussian irradiance profile of 10^{-5}m in diameter, defined by $\exp(-\pi)$ from the peak irradiance. If a lens with focal length 10mm is used to collimate the beam output from the fiber, what lens diameter should be used? Hint: You can assume that the lens is in the Fraunhofer region of the fiber output.

(b) (2pts) Design a test of temporal coherence using a Twyman-Green interferometer. The object of the test is to verify the width of the power spectrum. Draw the interferometer, label components, include what component(s) move during the measurement.

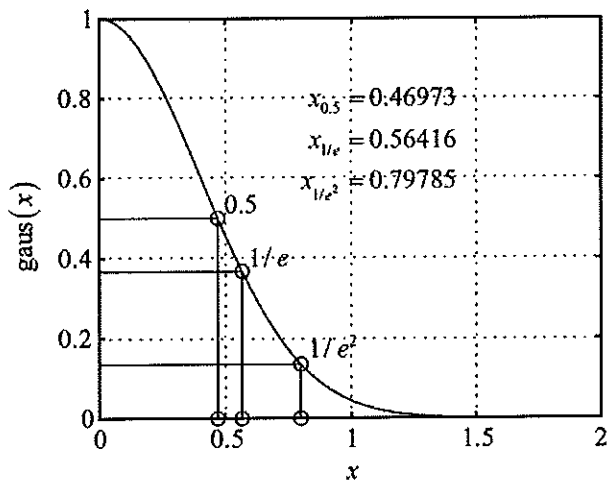
(c) (2pts) What is the minimum distance required to move one of the mirrors in (b) in order to test for the width of the power spectrum?

(d) (2 pts) If the distance from the collimating lens to one of the mirrors is 154mm, what is the Fresnel number of the beam at the mirror? Hint: Use your lens diameter from part (a).

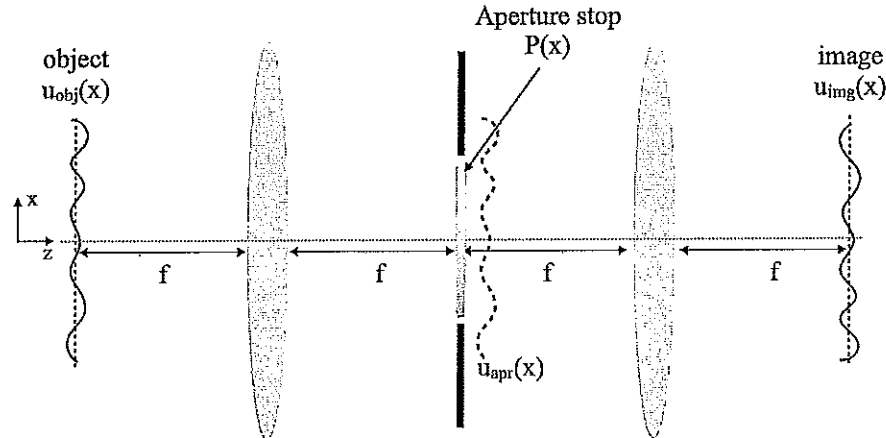
Useful Relations:

$$\text{gaus}(x) = \exp(-\pi x^2)$$

$$F_{\xi}[\text{gaus}(x)] = \text{gaus}(\xi)$$



Consider the following coherent 4-f imaging system with an aperture stop, with transmittance $P(x)$, placed between two identical lenses:



The object and image fields are specified by $u_{obj}(x)$ and $u_{img}(x)$ respectively. The front and back focal lengths of the two identical lenses are $f = 5\text{cm}$. In this system you may assume each lens is ideal, that is the lens has no aberrations and it is of infinite extent, so you can ignore any diffraction from the lens itself.

(a) Given a quasi-monochromatic ($\lambda = 500\text{nm}$) object field: $u_{obj}(x) = \cos^2(2\pi\xi_o x)$, derive the field $u_{apr}(x)$ in the aperture stop plane (located at $2f$) in the imaging system and find the location of the optical spots for the spatial frequency $\xi_o = 40\text{cycles/mm}$.

[3 points]

(b) Given an aperture stop of size $D = 10\text{mm}$ with a central obstruction of size $d = 1\text{mm}$: $P(x) = \text{rect}\left(\frac{x}{D}\right) - \text{rect}\left(\frac{x}{d}\right)$, compute the image field $u_{img}(x)$ for the object field specified in (a). Sketch the object and image irradiance patterns separately and comment on the similarities and differences. You may normalize the peak irradiance values in each case.

[4 points]

(c) Now consider a clear aperture stop of size $D = 10\text{mm}$: $P(x) = \text{rect}\left(\frac{x}{D}\right)$. What is the highest spatial frequency (ξ_{max}) in the object field that will pass through this imaging system as a function of illumination wavelength λ ? Compute ξ_{max} for $\lambda = 250\text{nm}$, $\lambda = 500\text{nm}$, and $\lambda = 1000\text{nm}$.

[3 points]

(a – 2 pts) For a one-dimensional coordinate system over the spatial coordinate x , write out the 1D *time-independent* Schrödinger equation for a ground-state wavefunction $\psi_g(x)$ of a potential $V(x)$. Let the energy eigenvalue associated with $\psi_g(x)$ be labeled E_g .

(b – 1 pt) If $\psi_g(x)$ is a Gaussian function, what type of potential is $V(x)$? You may give a name for $V(x)$ or a functional form.

(c – 3 pts) Consider a case that you have not encountered before: a potential well $V(x)$ for which the ground-state wavefunction is $\psi_g(x) = A e^{-x^4/a^4}$, where A and a are constants. Be sure to realize that $\psi_g(x)$ is *not* a Gaussian function. Using the 1D time-independent Schrödinger equation, determine the exact functional form of the potential well $V(x)$ for which $\psi_g(x) = A e^{-x^4/a^4}$ is the ground-state wavefunction. Let the ground-state energy eigenvalue E_g be equal to zero, which implies that the minimum of $V(x)$ is a negative value.

(d – 2 pts) Plot the potential $V(x)$ that you found in part (c). Make your plot reasonably accurate: the plot should show the significant characteristics of $V(x)$, with labels on important characteristic values.

(e – 2 pts) For $\psi_g(x) = A e^{-x^4/a^4}$, determine a numerical value for the constant A for the case $a = 10^{-6}$ m, such that $\psi_g(x)$ is properly normalized. Using your calculator to do this, give an answer accurate to three digits. You will need to use some of the information given below.

$$\int_{-\infty}^{\infty} e^{-u^2} du = 1.772\dots$$

$$\int_{-\infty}^{\infty} e^{-u^4} du = 1.813\dots$$

$$\int_{-\infty}^{\infty} e^{-u^6} du = 1.855\dots$$

$$\int_{-\infty}^{\infty} e^{-u^8} du = 1.883\dots$$

A thin glass cell of 1mm length contains a dilute gas of hydrogen atoms with number density $N = 1 \times 10^{10}\text{cm}^{-3}$. At time $t = 0$ a monochromatic laser field of the form $\vec{E} = \hat{z}\frac{1}{2}E_0e^{-i\omega t} + c.c.$ is turned on. The laser frequency is on resonance with the $n = 1$ to $n = 3$ transition. See the bottom of this page for additional information you may find useful.

(a) [2 points] When the laser intensity is very low, such that $I \ll I_{sat}$, the transmission of the beam is 99%, with only 1% being absorbed (scattered) by the atoms. Using this information on how much light is absorbed (scattered), calculate the on-resonance absorption cross section for this transition.

(b) [3 points] The field strength of the laser is now turned up to $E_0 = (\frac{\pi\hbar}{ea_0}) \times 10^8 \approx 3 \times 10^3 [\frac{J}{C\cdot m}]$. If the laser is turned on at time $t = 0$ and then turned off at time t_1 , calculate t_1 such that the laser puts every atom into a state with equal probability of being in the $1S$ state and the $3P$ ($m=0$) state. Provide a numeric answer.

(c) [3 points] Calculate the resulting *time-dependent* macroscopic polarization density starting at time t_1 , where $\vec{P} = N \times \langle \vec{d} \rangle$, where $\langle \vec{d} \rangle$ is the expectation value of the induced dipole moment. Provide a numeric answer that shows the magnitude and time dependence of \vec{P} .

(d) [2 points] Name 2 processes which could potentially cause decay of the macroscopic dipole moment.

The following expressions may be useful for these calculations:

$$\begin{aligned} a_0 &= 0.5 \times 10^{-10}(\text{m}) \\ e &= 1.6 \times 10^{-19}(\text{C}) \\ \hbar &= 1.05 \times 10^{-34}(\text{J}\cdot\text{s}) \end{aligned}$$

Cartesian Components of Angular Matrix Elements

$$\begin{aligned} \langle l=0, m=0 | \hat{r} | l=1, m=0 \rangle &= (0, 0, \sqrt{\frac{1}{3}}) \\ \langle l=0, m=0 | \hat{r} | l=1, m=1 \rangle &= (-\sqrt{\frac{1}{6}}, -i\sqrt{\frac{1}{6}}, 0) \\ \langle l=0, m=0 | \hat{r} | l=1, m=-1 \rangle &= (\sqrt{\frac{1}{6}}, -i\sqrt{\frac{1}{6}}, 0) \end{aligned}$$

Radial Coordinate Matrix Elements for Atomic Hydrogen

$$\begin{aligned} \langle 1S | r | 2P \rangle &= 1.29a_0 \\ \langle 1S | r | 3P \rangle &= 0.517a_0 \\ \langle 2S | r | 3P \rangle &= 3.07a_0 \\ \langle 2P | r | 3S \rangle &= 0.95a_0 \end{aligned}$$

Fall 2017 Comprehensive Exam
OPTI 507, Day 1

Consider a GaAs crystal. Choose the correct primitive translation vectors (note that this is for the real-space crystal, not reciprocal space) :

$$\vec{a} = \frac{1}{2}a(\hat{x} - \hat{y} + \hat{z}); \quad \vec{b} = \frac{1}{2}a(\hat{x} + \hat{y} - \hat{z}); \quad \vec{c} = \frac{1}{2}a(\hat{x} + \hat{y} + \hat{z})$$

$$\vec{a} = \frac{1}{2}a(\hat{x} + \hat{y}); \quad \vec{b} = \frac{1}{2}a(\hat{y} + \hat{z}); \quad \vec{c} = \frac{1}{2}a(\hat{x} + \hat{z})$$

$$\vec{a} = a\hat{x}; \quad \vec{b} = \frac{1}{2}a\hat{x} + \frac{\sqrt{3}}{2}a\hat{y}; \quad \vec{c} = c\hat{z}$$

Determine the Miller indices of the plane $z = a$, i.e. the plane parallel to the x-y plane at a distance a from the origin.

(10 points)

Fall 2017 Comprehensive Exam
OPTI 507, Day 2

Consider a laser containing a three-dimensional 2-band semiconductor with 2-fold spin degeneracy of each band (for simplicity assume $m_e = m_h = m_0$ where m_0 is the electron mass in vacuum, and Coulomb effects are neglected). Assume you have electrons in the conduction band and the holes in the valence band. The electron and hole densities are $n_e = n_h = 2 \times 10^{18} \text{ cm}^{-3}$. The distribution function of the conduction band electrons is assumed to be a zero-temperature Fermi function. Similarly, the distribution function of the holes is assumed to be zero-temperature Fermi function. Given a band gap of $E_g = 1.5 \text{ eV}$, calculate the energy of the gain-to-absorption cross-over and the gain bandwidth. Sketch the spectrum and indicate the band gap, cross-over, and bandwidth. (Hints: You need to determine k_F first. You may use $\hbar^2 / m_0 = 7.62 \times 10^{-16} \text{ eV cm}^2$.)

(10 points)

Fall 2017 Written Comprehensive Examination
OPTI-537

Answer the following questions related to solid-state physics and semiconductor detectors. All parts weighted as indicated.

(a) (10%) Derive the dispersion relation (hint: energy versus momentum expression) for a free-electron wave in one dimension. Generalize this expression to 3D.

(b) (10%) Write the expression that defines the relationship between the direct ($\mathbf{R} = l\mathbf{a} + m\mathbf{b} + n\mathbf{c}$) and reciprocal ($\mathbf{G} = o\mathbf{A} + p\mathbf{B} + q\mathbf{C}$) lattice vectors for a perfect crystal. What conditions must be satisfied for the direct lattice to be a 3D Bravais lattice? Define all terms.

(c) (10%) Explain what is meant by the 1st Brillouin zone, and write an expression for its volume for a cubic lattice with primitive direct lattice constants $a = b = c = 5 \text{ \AA}$.

(d) (20%) If a macroscopic crystal of the lattice structures in (b) above has dimensions of .5 mm x .5 mm x 1 mm in the $\hat{\mathbf{a}}$, $\hat{\mathbf{b}}$, and $\hat{\mathbf{c}}$ directions (which correspond to the x, y, and z directions, respectively), and we invoke Bloch's theorem and Born-Von Karman boundary conditions, what is the spacing between the allowed k_x , k_y , and k_z states? (Express your answer in the customary units of cm^{-1}).

(e) (10%) How many electrons fit inside a single band in the 1st Brillouin zone of this crystal?

(f) (20%, 10% each part) Then consider a 2D square lattice with primitive lattice constants $a = b = 5 \text{ \AA}$. Use the Ewald sphere construction to work out:

1) How many allowed elastic reflections (k_{out}) there are if light has an incident \mathbf{k} vector (in units of \AA^{-1})

$$\mathbf{k}_{inc} = \frac{3\pi}{5} \hat{\mathbf{A}} + \frac{\pi}{5} \hat{\mathbf{B}}$$

where $\hat{\mathbf{A}}$ and $\hat{\mathbf{B}}$ are the unit vectors in reciprocal space corresponding to directions $\hat{\mathbf{a}}$ and $\hat{\mathbf{b}}$ in real space.

2) Write the expressions for the allowed k_{out} vectors in terms of $\hat{\mathbf{A}}$ and $\hat{\mathbf{B}}$.

(g) (10%) Make a sketch of the basic structure of a CCD that shows how charge is stored following light absorption, and make a second diagram that explains how all of the data is read out from a 2D CCD imaging detector. Indicate the slow and fast clocks and where conversion to a voltage occurs.

(h) (10%) What charge transfer efficiency is required for a $2K \times 2K$ CCD (remember that the symbol K generally stands for 1024 in detector/electronics contexts) read out on a single edge if in the **worst case** 90% of the original charge in a pixel is to be retained when the read out process is complete?