WRITTEN PRELIM EXAM – FIRST DAY

Fall 2011

September 20, 2011
8:30 a.m. to 12:00 p.m.

Answer all six questions. Start each answer on a new page.

In the upper right hand corner of each sheet you hand in, put your code number (NO NAMES PLEASE) and the problem number. Staple together all sheets for a given problem.

Insert your answers into the manila envelope supplied and note which problems you have answered on the outside of the envelope.

The following are some helpful items:

$h = 6.625 \times 10^{-34}\ \mathrm{J \cdot s} = 4.134 \times 10^{-15}\ \mathrm{eV \cdot s}$

$e = 1.6 \times 10^{-19}\ \mathrm{C}$

$c = 3.0 \times 10^{8}\ \mathrm{m/s}$

$k_B = 1.38 \times 10^{-23}\ \mathrm{J/K}$

$\sigma = 5.67 \times 10^{-8}\ \mathrm{W/K^4 \cdot m^2}$

$\epsilon_0 = 8.85 \times 10^{-12}\ \mathrm{F/m}$

$\mu_0 = 1.26 \times 10^{-6}\ \mathrm{H/m}$

$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$

$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$

$2 \cos A \cos B = \cos(A - B) + \cos(A + B)$

$2 \sin A \sin B = \cos(A - B) - \cos(A + B)$

$2 \sin A \cos B = \sin(A + B) + \sin(A - B)$

$2 \cos A \sin B = \sin(A + B) - \sin(A - B)$

$\sin 2A = 2 \sin A \cos A$

$\cos 2A = 2 \cos^2 A - 1$

$\cos 2A = 1 - 2 \sin^2 A$

$\sinh x = \frac{1}{2}\left(e^{x} - e^{-x}\right)$

$\cosh x = \frac{1}{2}\left(e^{x} + e^{-x}\right)$

$\nabla(\phi + \psi) = \nabla\phi + \nabla\psi$

$\nabla\phi\psi = \phi\nabla\psi + \psi\nabla\phi$

$\nabla \cdot (\mathbf{F} + \mathbf{G}) = \nabla \cdot \mathbf{F} + \nabla \cdot \mathbf{G}$

$\nabla \times (\mathbf{F} + \mathbf{G}) = \nabla \times \mathbf{F} + \nabla \times \mathbf{G}$

$\nabla(\mathbf{F} \cdot \mathbf{G}) = (\mathbf{F} \cdot \nabla)\mathbf{G} + (\mathbf{G} \cdot \nabla)\mathbf{F} + \mathbf{F} \times (\nabla \times \mathbf{G}) + \mathbf{G} \times (\nabla \times \mathbf{F})$

$\nabla \cdot (\phi\mathbf{F}) = \phi(\nabla \cdot \mathbf{F}) + \mathbf{F} \cdot \nabla\phi$

$\nabla \cdot (\mathbf{F} \times \mathbf{G}) = \mathbf{G} \cdot \nabla \times \mathbf{F} - \mathbf{F} \cdot \nabla \times \mathbf{G}$

$\nabla \cdot (\nabla \times \mathbf{F}) = 0$

$\nabla \times (\phi\mathbf{F}) = \phi(\nabla \times \mathbf{F}) + \nabla\phi \times \mathbf{F}$

$\nabla \times (\mathbf{F} \times \mathbf{G}) = \mathbf{F}(\nabla \cdot \mathbf{G}) - \mathbf{G}(\nabla \cdot \mathbf{F}) + (\mathbf{G} \cdot \nabla)\mathbf{F} - (\mathbf{F} \cdot \nabla)\mathbf{G}$

$\nabla \times (\nabla \times \mathbf{F}) = \nabla(\nabla \cdot \mathbf{F}) - \nabla^2\mathbf{F}$

$\nabla \times \nabla\phi = 0$

$\oint_S (\mathbf{F} \cdot \mathbf{n})\, da = \int_V (\nabla \cdot \mathbf{F})\, d^3x$

$\oint_C \mathbf{F} \cdot d\ell = \int_S (\nabla \times \mathbf{F}) \cdot \mathbf{n}\, da$

$\oint_S \phi\mathbf{n}\, da = \int_V \nabla\phi\, d^3x$

$\oint_S \mathbf{F}(\mathbf{G} \cdot \mathbf{n})\, da = \int_V [\mathbf{F}(\nabla \cdot \mathbf{G}) + (\mathbf{G} \cdot \nabla)\mathbf{F}]\, d^3x$

$\oint_S (\mathbf{n} \times \mathbf{F})\, da = \int_V (\nabla \times \mathbf{F})\, d^3x$

Fall 2011 Comprehensive Exam
OPTI 507

Consider a cubic crystal with lattice constant a and primitive translation vectors $\vec{a} = a\,\hat{x},\quad \vec{b} = a\,\hat{y},\quad \vec{c} = a\,\hat{z}$.

(a) Find the Miller indices $(h\,k\,l)$ for a plane that intercepts the axes at $3\vec{a}, 4\vec{b}$ and $2\vec{c}$.

(3 points)

(b) Write the reciprocal lattice vector $\vec{G} = h\,\vec{A} + k\,\vec{B} + l\,\vec{C}$ in Cartesian coordinates (this includes determining $\vec{A}, \vec{B}, \vec{C}$ and the unit cell volume) and show that it is parallel to the vector that corresponds to the direction $[h\,k\,l]$, i.e. the vector $\vec{d} = h\,\vec{a} + k\,\vec{b} + l\,\vec{c}$. Determine the proportionality factor f in $\vec{G} = f\,\vec{d}$. Is the direction $[h\,k\,l]$ parallel to $\vec{G} = h\,\vec{A} + k\,\vec{B} + l\,\vec{C}$ in all, including non-cubic, crystal structures?

(7 points)

#1

Fall 2011 Written Comprehensive Examination
OPTI-537

Answer the following questions related to diffraction theory.

Explain, using equations and diagrams as necessary, the approximations and steps made in going from the full Rayleigh-Sommerfeld diffraction formula to Fresnel diffraction to Fraunhofer diffraction. Discuss the domains where each is applicable.

$$u_z(\mathbf{r}) = -\frac{1}{2\pi}\int d^2r_0\, u_0(\mathbf{r}_0)\,\frac{\partial}{\partial z}\frac{e^{ikR}}{R} \qquad R^2 = |\mathbf{r}-\mathbf{r}_0|^2$$

(a) (30%)

$$u_z(\mathbf{r}) = -\frac{1}{2\pi}\int d^2r_0\, u_0(\mathbf{r}_0)\left(ik - \frac{1}{R}\right)\frac{z}{R}\frac{e^{ik\sqrt{|\mathbf{r}-\mathbf{r}_0|^2+z^2}}}{\sqrt{|\mathbf{r}-\mathbf{r}_0|^2+z^2}}$$

(b) (30%)

$$u_z(\mathbf{r}) \approx \frac{e^{ikz}}{i\lambda z}\int d^2r_0\, u_0(\mathbf{r}_0)\, e^{i\frac{\pi}{\lambda z}|\mathbf{r}-\mathbf{r}_0|^2}$$

(c) (40%)

$$u_z(\mathbf{r}) \approx \frac{e^{ikz}e^{i\frac{\pi r^2}{\lambda z}}}{i\lambda z}\,\mathcal{F}_2\{u_0(\mathbf{r}_0)\}_{\boldsymbol{\rho}=\mathbf{r}/\lambda z}$$

Fall 2011 Written Comprehensive Exam
Opti 503

You have a focal plane array (FPA) with 10 micron pixel center-to-center distance that you plan to use to image in an optical system operating at a wavelength of 500 nm with incoherent illumination. You will be using the system with a 50-mm focal length lens, and you may assume that the lens is nearly diffraction-limited.

A. What is the fastest f/# that you can operate at in order to guarantee alias-free sampling of the image on the FPA? (3 points)

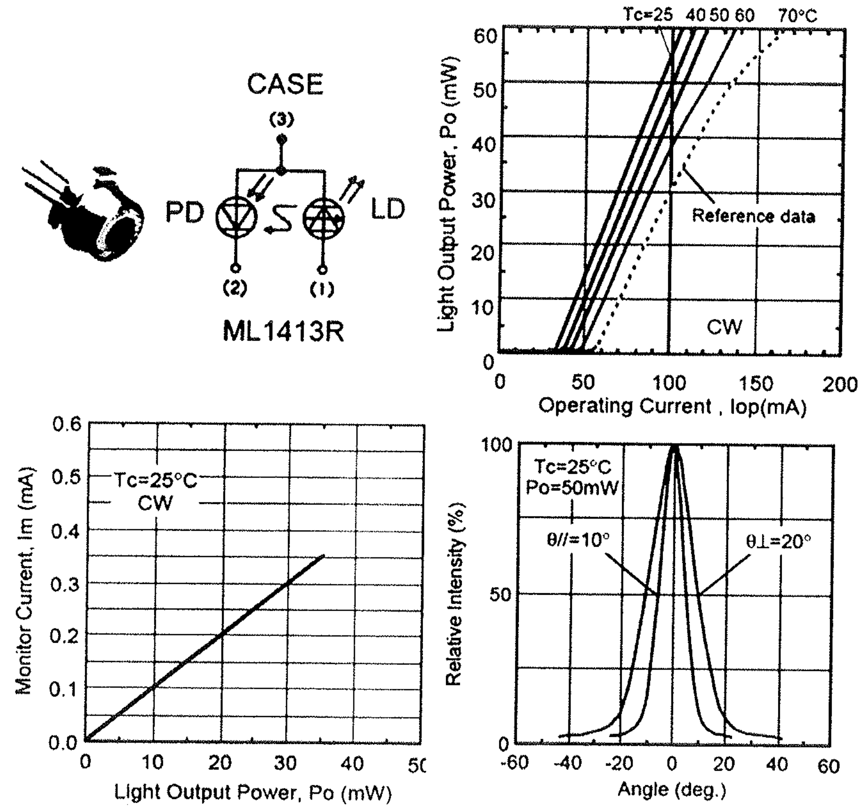
B. Now assume that the system is operating at f/10 with a pixel fill factor that is close to 100% (that is, the pixels are square and approximately 10 microns in width), sketch the MTF for the system that includes both the effects of the lens and the effects of the pixels (at this point ignore the aliasing effects that would be present in the system). (3 points)

C. How does the MTF change as the pixel active area get smaller, while maintaining the same center-to-center distance (pixel pitch)? (2 points)

D. Are there any practical considerations that make reducing the size of the pixels as we did in part C undesirable? (2 points)

Fall 2011 Written Comprehensive Exam
OPTI 510

Semiconductor pn junction as laser and detector



The above data describe the properties of a high power AlGaInP laser with integrated photodiode. The peak emission wavelength is 685nm.

(a) Depending on the material, device design and operating condition, a p-n junction can be used as a semiconductor light source or a semiconductor light detector. Describe two different characteristics or operating requirements of the p-n junction in the case of light source and in the case of a detector. (2 points)

(b) A picture and a diagram of a 3 terminal laser diode with detector are shown above. Draw a circuit diagram showing how you will bias the 3 terminal device to drive the laser diode and to read out the detector signal? You can use a voltage source and resistor(s). (2 points)

(c) From the graph, calculate the power-conversion efficiency of the laser diode at T=25C? (2 points)

(d) Assuming 5% of the output light goes into the detector, calculate the quantum efficiency of the detector at T=25C. (2 points)

(e) Is the diode a VCSEL or a side emitter? (2 points)

Fall 2011 Written Comprehensive Examination
OPTI-536

a. (2 points) The relaxed human eye has an optical power of approximately 58.6 diopters. What are values for the front and rear focal lengths of the human eye? You can assume the index of refraction of the media (vitreous humor) inside the human eye is 1.336
b. (2 points) You can assume the power of 58.6 diopters is for a relaxed eye imaging an object at infinity onto the retina. How much would the optical power of the eye have to change to allow the eye to focus on an object located at the near point of 250 mm in front of the front principal plane of the eye?
c. (1 point) If a thin lens is placed at the front focal plane of the relaxed eye, does it change the optical power of the combined lens eye imaging system?
d. (1 point) Based on your answer to part c, explain how such a lens allows the combined system (lens and relaxed eye) to image the object at the 250 mm near point object distance.
e. (2 points) If the eye were a diffraction limited incoherent imaging system with a pupil diameter of 2 mm, what is the maximum possible spatial frequency of an image on the retina. You can assume a wavelength of 500 nm and, that the relaxed eye is imaging an object at infinity, and that the pupil is located at the rear principal plane for this calculation.
f. (2 points) If you were designing an artificial retina and wanted to avoid any aliasing in the detected image, what would the maximum detector spacing you could have on your detector device?

Be sure to show your work in such a way that the equations used are clearly shown in order to get partial credit if you make a numerical or conceptual mistake in another part of the problem.

Fall 2011 Written Comprehensive Exam
OPTI 511R

Consider a laser composed of a stable linear cavity with 2 mirrors each having power reflectivity coefficients $R = 0.98$ and a homogeneously broadened 10 cm long gain medium. The gain medium is described by a 3-level system as diagramed below with population densities N_1, N_2, N_3. $N_T = 1 \times 10^{19} cm^{-3}$ is the total population density of this closed 3-level system. The spontaneous decay rates between dipole allowed transitions (Γ_{ij}) are also shown. An external pumping rate P is required for population inversion, where P_T is the pumping rate required to reach the lasing threshold on the 2-1 transition. Recall that the gain coefficient for the 2-1 transition can be written as: $\gamma = \frac{\delta N \cdot \sigma_0}{1+I/I_{sat}+4(\Delta/\Gamma_{21})^2}$, where the resonant absorption cross section for this gain medium is $\sigma_0 = 3 \times 10^{-20} cm^2$.

(a) (2 points) Argue why, based on the selection rules for dipole allowed transitions, there is no spontaneous emission between levels 3-1 in this system.

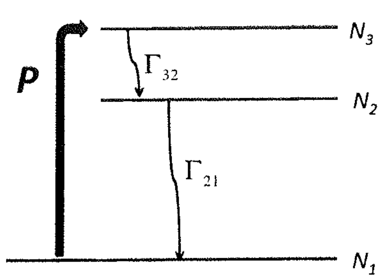
(b) (1 points) Write down the set of coupled rate equations for the 3-level system assuming $P < P_T$.

(c) (2 points) Assuming $\Gamma_{32} >> P$, derive the expression for the steady-state population inversion density on the 2-1 transition, $N_2 - N_1$, assuming $P < P_T$.

(d) (1 point) What is the minimum pumping rate needed to have gain rather than absorption of light resonant with the 2-1 transition?

(e) (2 points) Assume a shutter is momentarily placed inside the laser cavity to prevent it from lasing. If $P = 2\Gamma_{21}$, make a detailed plot of the gain coefficient versus detuning for the 2-1 transition. Be sure to indicate on the plot the numeric value of the gain coefficient at $\Delta = 0$ and the full-width at half maximum in terms of parameters defined in this problem.

(f) (2 points) Assuming $P = 2\Gamma_{21}$ and with shutter opened, is it possible for the laser to operate? Justify you answer.



Fall 2011 Written Comprehensive Exam
OPTI 544

Consider in the following a two-level atom with ground state $|1\rangle$, excited state $|2\rangle$, transition frequency ω_0, and a real-valued dipole matrix element $\vec{p}_{12} = \vec{p}_{21}$. The atom is driven by a real-valued classical field $\vec{E}(t) = \frac{1}{2}\vec{\varepsilon}E_0\left(e^{-i\omega t} + e^{i\omega t}\right)$, where $\vec{\varepsilon}$ is a unit vector parallel to $\vec{p}_{12}$. The state of the atom is described by a density operator ρ; in the Rotating Wave Approximation the evolution of the matrix elements of ρ are governed by the equations

$$\frac{d}{dt}\rho_{11} = A_{21}\rho_{22} - \frac{i}{2}\left(\chi\tilde{\rho}_{12} - \chi^*\tilde{\rho}_{21}\right), \qquad \frac{d}{dt}\rho_{22} = -A_{21}\rho_{22} + \frac{i}{2}\left(\chi\rho_{12} - \chi^*\rho_{21}\right)$$

$$\frac{d}{dt}\tilde{\rho}_{12} = \left(i\Delta - \frac{A_{21}}{2}\right)\tilde{\rho}_{12} + i\frac{\chi^*}{2}\left(\rho_{22} - \rho_{11}\right) = \left(\frac{d}{dt}\tilde{\rho}_{21}\right)^*$$

(a) *(2.5 pts)* Find the steady state value of the coherences $\tilde{\rho}_{12}$, $\tilde{\rho}_{21}$ in terms of ρ_{11}, ρ_{22}, χ, Δ and A_{21}.

(b) *(2.5 pts)* Give an expression for the steady state expectation value of the dipole $\langle\hat{\vec{p}}\rangle$ in terms of ρ_{11}, ρ_{22}, χ, Δ and A_{21}. Reminder: The relations between the coherences in the lab and rotating frames are $\rho_{12}(t) = \tilde{\rho}_{12}\, e^{i\omega t}$, $\rho_{21}(t) = \tilde{\rho}_{21}\, e^{-i\omega t}$.

(c) *(2.5 pts)* We define the complex polarizability via the relation, $\langle\hat{\vec{p}}\rangle^{(+)} = \alpha\,\vec{E}^{(+)}(t)$, where the superscript "(+)" indicates the positive frequency component of a real-valued quantity, e. g. $\vec{E}^{(+)}(t) = \frac{1}{2}\vec{\varepsilon}E_0 e^{-i\omega t}$. In general, α will depend on the inversion $\rho_{22} - \rho_{11}$. Find an expression for α in the low saturation limit, $|\chi| << A_{21}$ and/or $|\chi| << |\Delta|$. Hint: $\vec{p}_{12} = (\vec{p}_{12}\cdot\vec{\varepsilon})\,\vec{\varepsilon}$

(d) *(2.5 pts)* What happens to the complex index of refraction in the limit $|\chi| >> |\Delta|, A_{21}$? What does this imply for the absorption and dispersion seen by a field propagating through a medium composed of such atoms?

Fall 2011 Comprehensive Exam

Question 505R

Consider a Twyman-Green interferometer that is illuminated by a pulsed laser with the electric field

$$U(z,t) = A\,\mathrm{gaus}\left(\frac{t}{\tau}\right)e^{j(kz-\omega t)} \,,$$

where t is time, $k = 2\pi/\lambda$, τ is the pulse width, ω is radian frequency and z is distance. Only one pulse if analyzed for simplicity. The beam splitter divides the wavefront amplitude equally, and a delay Δ is introduced between the two interfering waves by adjusting the length of one arm of the interferometer.

a) (2.5pts) Write an expression that describes the field at the detector, assuming that no tilt is introduced between the interfering plane waves and the beam arriving at the detector from one arm can be written as

$$U_1(z,t) = CA\,\mathrm{gaus}\left(\frac{t}{\tau}\right)e^{j(kz-\omega t)} \,,$$

where C is a constant.

b) (2.5pts) Write an expression for the square magnitude of the total electric field and simplify.

c) (2.5pts) Find an expression or a quantity that is proportional to irradiance, which is the integral over time of (b). Simplify.

d) (2.5pts) This device is called a field autocorrelator, and it is sometimes used to measure very short laser pulse widths. Describe how pulse width τ can be measured by varying Δ.

You may find the following information useful:

$$\mathrm{gaus}\left(\frac{x}{b}\right) = e^{-\pi(x/b)^2}$$

$$\mathbf{F}_\xi\left[\mathrm{gaus}(x)\right] = \mathrm{gaus}(\xi)$$

$$\int_\infty \mathrm{gaus}\left(\frac{x}{b}\right)dx = |b|$$

Fall 2011 Written Comprehensive Exam
Opti 546

This problem involves a plane-wave $E_0 e^{ikz}$ propagating along the z-axis that impinges upon a screen at $z = 0$ with field transmission $t(x')$, resulting in a field just after the screen $E(x', z = 0) = E_0 t(x')$. Here we assume the screen and field are homogeneous along the y-axis and restrict the analysis to one transverse dimension. Then within the Fresnel approximation the field in the Fraunhofer region a distance L beyond the screen is given by

$$E(x, L) = \int_{-\infty}^{\infty} dx' E(x', 0) e^{-\frac{ikxx'}{L}},$$

where for simplicity a prefactor multiplying the integral has been omitted. Equal points are allocated to each part of this question.

(a) First consider the case of a single aperture with transmission $t(x') = rect\left(\frac{(x'-x_0)}{a}\right)$, x_0 being the position of the aperture center and 'a' the aperture width. Here $rect(s)$ is the standard rectangular function with

$$rect(s) = \begin{cases} 0, & |s| > 1/2 \\ 1/2, & |s| = 1/2 \\ 1, & |s| < 1/2 \end{cases}$$

By directly evaluating the diffraction integral above derive an expression for the diffracted field $E(x, L)$ in the Fraunhofer region.

(b) Next consider the case with two apertures whose centers are separated by a distance d with transmission $t(x') = \left[rect\left(\frac{(x'+d/2)}{a}\right) + e^{i\phi} rect\left(\frac{(x'-d/2)}{a}\right)\right]$, with ϕ the relative phase between the two apertures. Use your result from part (a) to obtain an expression for the resulting transverse intensity profile $|E(x, L)|^2$ for the case $\phi = \pi$.

(c) Using your result for $|E(x, L)|^2$ from part (b) derive expressions for those transverse coordinates where the field intensity must be zero.

(d) Assuming $d = 4a$, and using your result for $|E(x, L)|^2$ from part (b), provide a sketch of the normalized intensity $\frac{|E|^2}{4a^2|E_0|^2}$ versus the scaled coordinate $R = \frac{kax}{2L}$ in the range $R = [-\pi, \pi]$. Make sure to indicate the key features in your plot such as the position of the intensity zeros.

Fall 2011 Written Comprehensive Exam
Opti 502

Two thin lenses of equal power (ϕ) are separated by a distance, t, in air. Assume paraxial optics.

a. Using the paraxial raytrace equations, derive the expression for the total system power (ϕ_T) as a function of t. (4 points)

b. What is the total system power when the two thin lenses are in contact? (1 point)

c. Derive an expression for the location of the rear principal point of the system (P′) relative to the second thin lens. (3 points)

d. What is the total system power when the two thin lenses are separated by the focal length of one of the thin lenses? Where are locations of the rear focal point and the rear principal plane? (1 point)

e. What is the total system power when the two thin lenses are separated by twice the focal length of one of the thin lenses? Where are locations of the rear focal point and the rear principal plane? (1 point)

WRITTEN PRELIM EXAM – SECOND DAY

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$\nabla(\mathbf{F} \cdot \mathbf{G}) = (\mathbf{F} \cdot \nabla)\mathbf{G} + (\mathbf{G} \cdot \nabla)\mathbf{F} + \mathbf{F} \times (\nabla \times \mathbf{G}) + \mathbf{G} \times (\nabla \times \mathbf{F})$

$\nabla \cdot (\phi\mathbf{F}) = \phi(\nabla \cdot \mathbf{F}) + \mathbf{F} \cdot \nabla\phi$

$\nabla \cdot (\mathbf{F} \times \mathbf{G}) = \mathbf{G} \cdot \nabla \times \mathbf{F} - \mathbf{F} \cdot \nabla \times \mathbf{G}$

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$\nabla \times (\mathbf{F} \times \mathbf{G}) = \mathbf{F}(\nabla \cdot \mathbf{G}) - \mathbf{G}(\nabla \cdot \mathbf{F}) + (\mathbf{G} \cdot \nabla)\mathbf{F} - (\mathbf{F} \cdot \nabla)\mathbf{G}$

$\nabla \times (\nabla \times \mathbf{F}) = \nabla(\nabla \cdot \mathbf{F}) - \nabla^2\mathbf{F}$

$\nabla \times \nabla\phi = 0$

$\oint_{\mathrm{S}} (\mathbf{F} \cdot \mathbf{n})\, da = \int_{\mathrm{V}} (\nabla \cdot \mathbf{F})\, d^3x$

$\oint_{\mathrm{C}} \mathbf{F} \cdot d\ell = \int_{\mathrm{S}} (\nabla \times \mathbf{F}) \cdot \mathbf{n}\, da$

$\oint_{\mathrm{S}} \phi\mathbf{n}\, da = \int_{\mathrm{V}} \nabla\phi\, d^3x$

$\oint_{\mathrm{S}} \mathbf{F}(\mathbf{G} \cdot \mathbf{n})\, da = \int_{\mathrm{V}} [\mathbf{F}(\nabla \cdot \mathbf{G}) + (\mathbf{G} \cdot \nabla)\mathbf{F}]\, d^3x$

$\oint_{\mathrm{S}} (\mathbf{n} \times \mathbf{F})\, da = \int_{\mathrm{V}} (\nabla \times \mathbf{F})\, d^3x$

Fall 2011 Comprehensive Exam
OPTI 507

(a) The Wannier equation for the relative motion of an exciton is

$$\left\{-\frac{\hbar^2}{2m_r}\nabla_r^2 - \frac{e^2}{\varepsilon_0 r}\right\}\varphi(\vec{r}) = E_r\,\varphi(\vec{r})\,.$$

Write m_r in terms of the electron and hole mass, and write E_r in terms of the quantum numbers and the 1s exciton binding energy E_B . Give an approximate value for the binding energy of the 1s exciton in GaAs, and determine the corresponding binding energies of the 2s and 3s excitons.

(4 points)

(b) Write the Schroedinger equation for the center-of-mass motion and give the solution for the center-of-mass wave function $g(\vec{R})$ (no normalization factor is requested). Clearly specify the quantum numbers and eigenenergies for the center of mass motion in terms of the electron and hole mass. Give an estimate of the magnitude of the center-of-mass wave vector of an optically excited exciton in GaAs. You need to justify your estimate clearly in order to get credit. This estimate should just be an order-of-magnitude estimate and should not contain any consideration of polariton effects. Sketch the dispersion relation for the center-of-mass energy of the 1s, 2s, and 3s excitons, clearly indicating the bandgap E_G and the binding energy of the 1s exciton.

(6 points)

Fall 2011 Written Comprehensive Examination
OPTI-537

Answer the following questions related to semiconductor detectors.

(a) (10%) What is the definition of a Bravais lattice? What is a primitive lattice vector? Write an expression that expresses the equivalence of locations in a real-space Bravais lattice.

(b) (10%) What is a reciprocal lattice? What is the relationship that defines reciprocal lattice vectors in terms of real-space lattice vectors?

(c) (10%) What is a Brillouin zone? What are the Brillouin zone boundaries for an orthorhombic real-space lattice with primitive vectors **a**, **b**, **c** ($|\mathbf{a}| \neq |\mathbf{b}| \neq |\mathbf{c}|$, $\mathbf{a} \perp \mathbf{b} \perp \mathbf{c}$)?

(d) (20%) What are energy bands and why do they arise? What is a bandgap? What is the importance of the bandgap in determining the properties of a semiconductor? Sketch a band diagram for the nearly-free electron model. Using this diagram, explain why a metal conducts electricity at absolute zero while a semiconductor doesn't.

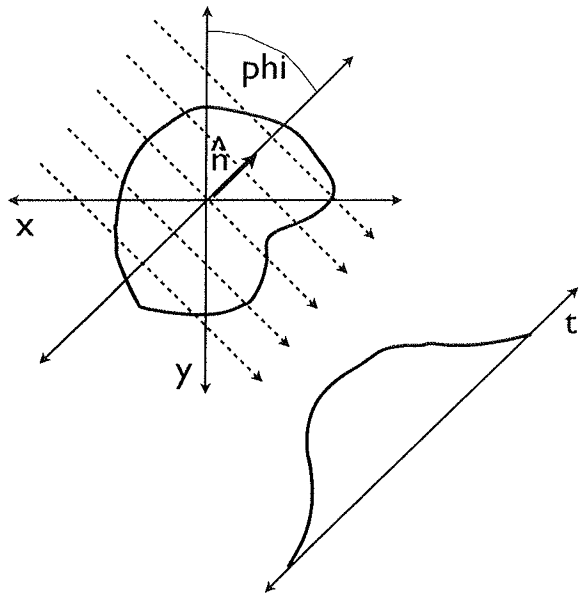
(e) (10%) Explain the difference between a direct versus indirect bandgap material. What kind is Si?

(f) (10%) What is the definition of the Fermi level? Write the expression for Fermi-Dirac Statistics and explain what it describes?

(g) (10%) Explain why donor and acceptor impurities are added to Si. Sketch the location (in an energy versus position band diagram) of donor and acceptor dopant states relative to the conduction and valence bands.

(h) (20%) Draw the band structure of the PN junction under reverse bias, identify the depletion region, and explain its significance for photodetection.

Fall 2011 Written Comprehensive Examination
OPTI-536



Consider a two-dimensional object $f(x, y)$. This object could, for example, represent the 2D map of attenuation coefficients in X-ray CT. The 2D Radon transform (RT) maps $f(x, y)$ to a function $p_\phi(t)$ where t is a distance along a projection axis, and ϕ is an angle of projection. (See the above figure for a complete description of the geometry).

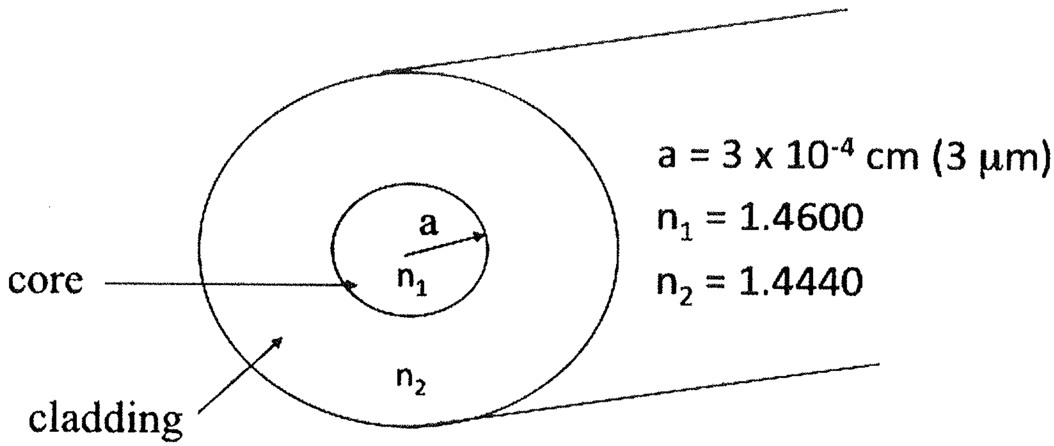
1. (10%) Write a mathematical expression for the Radon Transform.
2. (10%) Describe any symmetries that exist in this transform.
3. (20%) Derive the Central-Slice Theorem.

The goal of image reconstruction is to take measured projection data $p_\phi(t)$ and "reconstruct" the original object distribution $f(x, y)$. For the following questions, I would like a discussion of some methods for accomplishing this task. Math is not necessary but may be used if you find it easier.

4. (20%) Describe how the Central-Slice Theorem could be used to reconstruct the object $f(x, y)$.
5. (20%) Describe back projection (without any filtering) and discuss why this method DOES NOT work for image reconstruction.
6. (20%) Describe filtered back projection and sketch the shape of the filter used (sketch can be in the Fourier domain or the spatial domain).

Fall 2011 Written Comprehensive Exam
OPTI 510

Consider the step index fiber below where the core and cladding are dielectric materials with refractive indices n_1 and n_2, respectively, at 1550nm wavelength.



a) Write the full Helmholtz equation in the coordinate system appropriate for the optical fiber geometry (2 points).

b) Calculate the V parameter and the numerical aperture (NA) for this fiber (at 1550nm). Indicate whether the fiber is single-mode or multimode. (4 points).

Now suppose that dielectric medium 2 is changed to being a perfectly reflecting and lossless metal so that it acts like a perfect mirror for light guided in the core.

c) Describe the boundary condition on the electric field at $r = a$ and give the general form of the electric field's radial dependence in the two regions $r < a$ and $r > a$. (2 points)

d) For this perfect mirror fiber, will there always be at least one guided mode for any value of a ($a > 0$)? Why or why not? (2 points)

Fall 2011 Written Comprehensive Exam
Opti 503

Spherical aberration
Each part is worth 1 point.

The achromatic cemented doublet of a telescope has 10 waves (λ=0.0005 mm) of fourth-order spherical aberration and works at f/10.

a) Provide a wave aberration fan. Properly label the axes and draw.
b) Provide a transverse ray aberration fan. Properly label the axes and draw.
c) How much spherical aberration is there for a field of view of 1 degree?
d) If the stop aperture is closed as to change the F-number to f/20, how much does the spherical aberration wavefront error change?
e) Describe how the spherical aberration at f/10 in the doublet can be corrected.
f) At f/10, provide the distance from paraxial focus to the marginal ray intersection with the optical axis?
g) At f/10, provide the distance from paraxial focus that is required to minimize the rms radial spot size?
h) At f/10, provide the distance from paraxial focus that is required to minimize the rms wavefront error?
i) Provide the defocus in parts 7 and 8 as the defocus wavefront change.
j) How much light is lost when the aperture stop is closed to f/20?

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Opti 544

Consider two identical two-level atoms, each with a resonance frequency ω_0. Each atom is weakly coupled to an optical cavity mode also of frequency ω_0, but the atoms are not directly coupled to each other. Neglect spontaneous emission in this problem. In the absence of atom-photon coupling, the system's energy eigenstates are $|n_1\ n_p\ n_2\rangle$, where n_1=0 if atom 1 is in the ground state, n_1=1 if it is in the excited state. Similarly, n_2=0 or n_2=1 (atom 2 in ground or excited state, respectively). The cavity photon number is n_p.

At time t=0, atom 1 is in its excited state, n_p=0, and atom 2 is in its ground state. Hence the quantum state of the system is $|100\rangle$. With the atoms coupled to the cavity mode, but not to each other, the only bare states that can be populated at later times are $|010\rangle$ and $|001\rangle$. We thus limit this problem to the state space spanned by these three state vectors. For a suitable choice of energy scale, and for a real coupling strength Ω, the system Hamiltonian takes the form

$$H = \frac{\hbar\Omega}{\sqrt{2}}\left(\sigma_1 a^\dagger + \sigma_2 a^\dagger + \sigma_1^\dagger a + \sigma_2^\dagger a\right)$$

defined in terms of atom and photon raising and lowering operators: σ_1 is the lowering operator for atom 1, σ_2 is the lowering operator for atom 2, and a is the photon annihilation operator. For example $\sigma_1 a^\dagger|100\rangle = |010\rangle$ and $\sigma_2 a^\dagger|100\rangle = 0$.

(a) *(2 pts)* Using the $\{|100\rangle, |010\rangle, |001\rangle\}$ representation, write the 3x3 matrix corresponding to H. Remember, photon number is the middle number in the kets!

(b) *(2 pts)* List the eigenvalues of H. *Hint*: The eigenvectors of the matrix

$$M = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} \quad \text{are} \quad \hat{e}_1 = \begin{bmatrix} \frac{1}{2} \\ \frac{1}{\sqrt{2}} \\ \frac{1}{2} \end{bmatrix}, \ \hat{e}_2 = \begin{bmatrix} \frac{1}{\sqrt{2}} \\ 0 \\ -\frac{1}{\sqrt{2}} \end{bmatrix}, \ \text{and } \hat{e}_3 = \begin{bmatrix} \frac{1}{2} \\ -\frac{1}{\sqrt{2}} \\ \frac{1}{2} \end{bmatrix}$$

(c) *(3 pts)* At t=0, the quantum state of the system is $|\Psi(t=0)\rangle = |100\rangle$. Derive an expression for the quantum state $|\Psi(t>0)\rangle$ in terms of the eigenstates of H (ie, the energy eigenstates of the *coupled* system).

(d) *(3 pts)* Based on your result from (c), determine the probability of finding one photon in the cavity mode and both atoms in their ground states at any time t>0.

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OPTI 511R

Consider a hydrogen atom at rest in the absence of any external fields in the following superposition state at some time t=0:

$\psi = \frac{1}{\sqrt{2}}[\psi_{100} + \psi_{310}]$, where the energy eigenstates are given by the usual ψ_{nlm} notation.

(a) (1 points) In this superposition state, what is the expectation value for the magnitude of total orbital angular momentum of the atom?

(b) (1 point) What is the probability that a single measurement of the magnitude of total orbital angular momentum will yield the value you found in part (a)?

(c) (2 points) Write the time dependent form of the wavefunction, $\Psi(t)$, for this superposition state.

(d) (5 points) Calculate the time dependent expectation value of the atomic dipole moment for this superposition state, $\langle \vec{d}(t) \rangle$, where $\vec{d} = -e\vec{r}$. Be sure to indicate the magnitude of the dipole moment and the numeric value of its oscillation frequency. Neglect the effects of spontaneous emission.

(e) (1 point) Describe the effect spontaneous emission would have on $\langle \vec{d}(t) \rangle$.

Some helpful expressions:

$\hbar \approx 7 \times 10^{-16}$ eV·s

Cartesian Components of Angular Matrix Elements
$\langle l=1, m=0 \,|\hat{\mathbf{r}}|\, l=0, m=0 \rangle = (0, 0, \sqrt{\tfrac{1}{3}})$
$\langle l=1, m=+1 \,|\hat{\mathbf{r}}|\, l=0, m=0 \rangle = (-\sqrt{\tfrac{1}{6}}, i\sqrt{\tfrac{1}{6}}, 0)$
$\langle l=1, m=-1 \,|\hat{\mathbf{r}}|\, l=0. m=0 \rangle = (\sqrt{\tfrac{1}{6}}, i\sqrt{\tfrac{1}{6}}, 0)$

Radial Matrix Element for Atomic Hydrogen
$\langle n=1, l=0 \,|r|\, n=2, l=1 \rangle = 1.29a_0$
$\langle n=1, l=0 \,|r|\, n=3, l=1 \rangle = 0.517a_0$

Fall 2011 Comprehensive Exam, Question 505R-MilsterB

A 2mm diameter hole is illuminated by an on-axis plane wave with λ=500nm and amplitude A V/m.

a) (2pts) Write an expression for the field transmitted through the hole and immediately after the hole.
b) (2pts) Write an expression for the irradiance distribution in air in the Fraunhofer region.
c) (2pts) What is the central zero-to-zero width of the distribution at a distance of 20m?
d) (2pts) Sketch the irradiance profile at a distance of 0.5m. Indicate the positions of zeros, if any.
e) (2pts) A p = 100μm period phase grating is placed over the hole, which has amplitude transmission function

$$t_{grat}(y_s) = \exp\left\{ j\pi \left[\text{rect}\left(\frac{y_s}{p}\right) * \frac{1}{2p}\text{comb}\left(\frac{y_s}{2p}\right)\right]\right\},$$

where * denotes one-dimensional convolution. Sketch irradiance profiles along $(x_0,0)$ and $(0,y_0)$ observed at a distance of 0.5m.

You may find the following information useful:

$$\text{somb}\left(\frac{r}{d}\right) = \frac{2J_1\left(\frac{\pi r}{d}\right)}{\frac{\pi r}{d}}$$

$$\boldsymbol{B}_\rho\left\{\text{cyl}\left(\frac{r}{D}\right)\right\} = \frac{\pi}{4}D^2\text{somb}(D\rho)$$

$$\text{somb}\left(\frac{r}{d}\right) = 0 \text{ at } r = 1.22, 2.23, 3.24, \ldots$$

Fall 2011 Written Comprehensive Exam
Opti 546

This problem deals with the properties of the Laguerre-Gaussian (LG) modes of optical resonators. Recall that the mode frequencies of a two-mirror optical resonator are given by

$$\nu_{p\ell q} = \frac{c}{2L}[q + \frac{1}{\pi}(2p + |\ell| + 1) \cdot \cos^{-1}(\sqrt{g_1 g_2})],$$

with $p = 0, 1, 2, \ldots, \ell = 0, \pm1, \pm2, \ldots$, and $q >> 1$. A mode with index ℓ has an associated spatial variation $\cos(\ell\theta)$, and mode stability requires that $0 \leq g_1 g_2 \leq 1$.

(a - 2pts) Suppose a concentric optical resonator has one mirror of curvature 1 m. Give numbers for the length of the cavity and the free spectral range of the cavity in GHz.

(b - 2pts) Prove that a concentric resonator is stable if the cavity length is slightly less than the precise cavity length required for a concentric cavity, but is unstable if the length is slightly larger.

(c - 2pts) Suppose that a two-mirror optical cavity has a frequency difference between any two adjacent radial modes, all other mode indices being fixed, that is equal to one-half of the free spectral range. If the two mirrors have the same radius of curvature R, what is the length L of the cavity relative to the mirror curvature?

(d - 2pts) For a confocal resonator consider the family of transverse modes with mode indices (p, ℓ) for which $(2p + |\ell|) = M$ is equal to an even integer M. Show that this family of transverse modes coincides in frequency with the mode with indices $(0, 0, q + M/2)$. (Confocal optical resonators are special due to this degeneracy).

(e - 2pts) Draw the transverse intensity spot patterns for the $LG_{p\ell}$ laser modes with $(p, \ell) = (0, 0), (2, 0), (0, 2)$.

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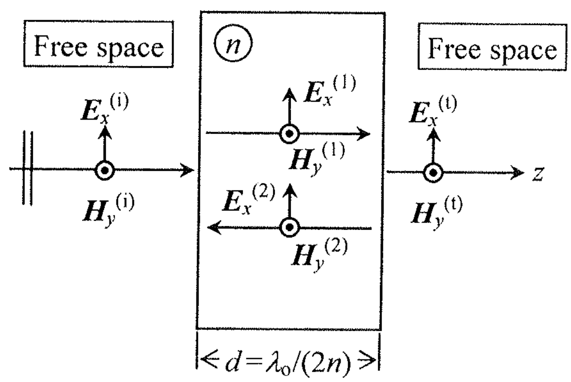
Opti 501

System of units: MKSA

It is a well-known fact that, at normal incidence, the reflectivity of a transparent dielectric slab of refractive index n and thickness $d=\lambda_0/(2n)$ is precisely equal to zero; here $\lambda_0=2\pi c/\omega_0$ is the vacuum wavelength of the incident beam, which, as shown in the figure, is a homogeneous, monochromatic plane-wave of frequency ω_0. (You may assume that the incident beam is linearly polarized along the x-axis.)



(4 pt) a) Using the aforementioned fact, determine the E- and H-fields of the forward- and backward-propagating plane-waves inside the slab, as well as the E- and H-fields of the transmitted beam.

(4 pt) b) Determine the *time-averaged* Poynting vector inside the slab, and show that the rate of flow of optical energy per unit cross-sectional area inside the slab is the same as that of the incident beam, and also the same as that of the transmitted beam.

(2 pts) c) In what ways will the results obtained in parts (a) and (b) change, if the thickness d of the slab happens to be an integer-multiple of $\lambda_0/(2n)$, that is, if $d=m\lambda_0/(2n)$, where $m\neq 1$ is an arbitrary integer?

Fall 2011 Written Comprehensive Exam
Opti 502

An infinity-corrected microscope consists of an objective lens, a tube lens and a simple eyepiece. The system is telecentric in object space and the objective lens has a focal length of 10 mm. The object space NA is 0.1. The overall system visual magnification is 100X and the eyepiece has a magnifying power of 10X. The separation between the objective lens and the tube lens is 160 mm. Assume all lenses are thin lenses in air.

a) Draw a diagram of the microscope indicating the location and size of the stop, the focal lengths of the tube lens and the eye lens, and the eye lens location. (3 points)

b) The maximum object diameter is 2.0 mm. What is the required diameter of the tube lens for the system to be unvignetted? (3 points)

c) Determine the location and size of the system exit pupil. (2 points)

d) A cube beamsplitter is to be inserted into the microscope in order to simultaneously capture the image on a 10 mm square image detector. Determine and sketch the required optical layout. Only one additional thin lens is to be used and its focal length must be provided. (2 points)